Connectivity in Wireless Sensor Networks

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Outline

- Introduction and motivation
- Connectivity analysis
- Link probability analysis
- Simulation results
- Conclusion and future work
Introduction and motivation

- Connectivity is a fundamental property and provides design reference for upper-layer protocols.
- Boolean disk model is too simple, real wireless channel is dynamic, with path loss, fading and shadowing.
- Investigate connectivity under more realistic channel model, study how various parameters impact the quality of connectivity.
Main idea

One-connectivity → Giant component

Non-isolation probability → Global property

Link probability ← Radio model

Local property
Connectivity analysis

- Node spatial distribution—homogenous Poisson point process

$N(A, \cdot)$ is the number of nodes in subarea $A$ which follows Poisson distribution with mean $\lambda(A)$:

$$P(N(A) = k) = \frac{\lambda(A)^k}{k!} e^{-\lambda(A)}, \text{ all } A \in \mathcal{F},$$  \hspace{1cm} (1)

with an expected value $E(N) = \lambda(A) = \rho(A)\|A\|$, $\rho$ and $\|A\|$ are node density and size of subarea $A$ respectively.

If $A_1, A_2, \ldots$ are disjoint sets then $N(A_1, \cdot), N(A_2, \cdot), \ldots$ are independent random variables:

$$P(N(A_1) = k_1 \land N(A_2) = k_2 \land \ldots \land N(A_n) = k_n) = \prod_{i=1}^{n} P(N(A_i) = k_i).$$  \hspace{1cm} (2)
Connectivity analysis

- One-connectivity probability $P(C)$ vs. node non-isolation probability $P(\bar{I})$
  
  - Node non-isolation is the necessary but non-sufficient condition of one-connectivity, thus $P(C) \leq P(\bar{I})$
  
  - Suppose the isolation of nodes to be independent events, we have:

    $$P(\bar{I}) = \sum_{k=0}^{\infty} P(\bar{I}|N = k) \cdot P(N = k)$$

    $$= \sum_{k=0}^{\infty} (1 - P(I))^k \cdot \frac{\lambda^k}{k!} e^{-\lambda}$$

    $$= e^{-\lambda P(I)} \sum_{k=0}^{\infty} \frac{[(1 - P(I))\lambda]^k}{k!} e^{-(1-P(I))\lambda}$$

    $$= e^{-\lambda P(I)}.$$
Connectivity analysis

- $P(I)$ is node isolation probability, $D$ is node degree, $D$ also follows Poisson distribution, we have:
  \[ P(I) = P(D = 0) = e^{-D_0}. \]
- $D_0$ can be calculated as
  \[
  D_0 = \rho \int_0^{2\pi} \int_0^{\infty} P(L|s) s \, ds \, d\phi \\
  = 2\pi \rho \int_\infty^{\infty} P(L|s) s \, ds.
  \]
  \[
  P(\bar{I}) = \exp(-\rho\|A\| \exp(-2\pi \rho \xi)), \quad \xi = \int_0^{\infty} P(L|s) s \, ds.
  \]
Link probability analysis

- Log-normal shadowing model

\[ P_r(s) = P_t - PL(s_0) - 10\eta \log\left(\frac{s}{s_0}\right) + \mathcal{N}(0, \sigma), \]

- Incorporating anisotropic property

\[ P_r(s) = P_t - (PL(s_0) + 10\eta \log\left(\frac{s}{s_0}\right)) \cdot K_i + \mathcal{N}(0, \sigma), \]

\[ K_i = \begin{cases} 
1, & \text{if } i = 0; \\
K_{i-1} + \text{rand} \cdot DOI, & \text{if } 0 < i < 360 \land i \in \mathbb{N}, \\
DOI = 0.01821. & 
\end{cases} \]
Link probability analysis

- PRR as a function of BER, taking NCFSK, Manchester encoding for example:
  \[
  \Psi(\gamma) = (1 - \beta_M(\gamma))^{8(2f-h)}, \quad \beta_M = \frac{1}{2}e^{-\frac{\gamma}{2}}, \quad \gamma = 10^{\gamma dB/10}.
  \]
  \[
  \Psi(\gamma) = (1 - \frac{1}{2}e^{-\frac{\gamma}{2}})^{8(2f-h)}.
  \]

- SNR can be obtained from the radio model
  \[
  \gamma(s) = P_r(s) - P_n,
  \]
  \[
  \mathcal{N}(\mu(s), \sigma) \quad \mu(s) = P_t - (PL(s_0) + 10\eta \log(\frac{s}{s_0})) \cdot K_i - P_n.
  \]

- Finally,
  \[
  \xi = \int_0^\infty P(L|s) s \, ds = \int_0^\infty \int_{-\infty}^{\infty} \Psi(\gamma) f(\gamma|s) \cdot s \, d\gamma \, ds.
  \]
Link probability analysis

\[
\Psi(\gamma) = \begin{cases} 
0, & \gamma \leq \gamma_0; \\
k_\psi \gamma + b_\psi, & \gamma_0 < \gamma < \gamma_1; \\
1, & \gamma > \gamma_1.
\end{cases}
\]

\[
k_\psi = \frac{0.9 - 0.1}{\gamma_1 - \gamma_0}, \quad b_\psi = \frac{0.1\gamma_1 - 0.9\gamma_0}{\gamma_1 - \gamma_0},
\]

\[
\gamma_0 = \Psi^{-1}(0.1), \quad \gamma_1 = \Psi^{-1}(0.9).
\]

\[
f(\gamma|s) = k_f(s) \gamma + b_f(s), \quad \gamma \in [\gamma_0, \gamma_1],
\]

\[
k_f(s) = \frac{f(\gamma_1|s) - f(\gamma_0|s)}{\gamma_1 - \gamma_0}, \quad b_f(s) = \frac{f(\gamma_0|s)\gamma_1 - f(\gamma_1|s)\gamma_0}{\gamma_1 - \gamma_0}
\]
Link probability analysis

\[ \xi = \int_0^\infty \int_{-\infty}^\infty \Psi(\gamma) f(\gamma|s) \cdot s \, d\gamma \, ds \]

\[ \approx \int_0^\infty \int_{\gamma_0}^\infty \Psi(\gamma) f(\gamma|s) \cdot s \, d\gamma \, ds \]

\[ \approx \int_0^\infty \int_{\gamma_0}^{\gamma_1} (k_{\gamma}\gamma + b_{\psi})(k_f(s)\gamma + b_f(s)) \cdot s \, d\gamma \, ds + \int_0^\infty (1 - \Phi(\frac{\gamma_1 - \mu(s)}{\sigma})) \cdot s \, ds \]

\[ \approx \int_0^{d_1} \int_{\gamma_0}^{\gamma_1} (k_{\gamma}\gamma + b_{\psi})(k_f(s)\gamma + b_f(s)) \cdot s \, d\gamma \, ds + \int_0^{d_1} (1 - \Phi(\frac{\gamma_1 - \mu(s)}{\sigma})) \cdot s \, ds, \]
Simulation settings

- Select a subarea, eliminating border effect
- Baseline: $n=3, \sigma=3.5, P_t=-5\text{dBm}$, encoding method=Manchester, frame size=50 bytes, $|A|=20000m^2$ (modulation=NCFSK)
- Comparison: theoretical-simulation results of non-isolation prob.; one-connectivity vs. non-isolation prob.; giant component vs. one-connectivity; impact of different parameters.
n=3, n=3.5, P_t=5dBm, enc=Manchester, f=50B

(a) prob. 1-connectivity
(b) prob. non-isolation
(c) prob. non-isolation (analysis)
(d) largest component size
(e) prob. giant component

n=3, n=3.5, P_t=5dBm, enc=Manchester, f=50B, A=40000m^2

(f) prob. 1-connectivity
(g) prob. non-isolation
(h) prob. non-isolation (analysis)
(i) largest component size
(j) prob. giant component

n=3, n=3.5, P_t=5dBm, enc=Manchester, f=100B

(k) prob. 1-connectivity
(l) prob. non-isolation
(m) prob. non-isolation (analysis)
(n) largest component size
(o) prob. giant component
Discussion

- As node density increases, the transition from low connectivity and the appearance of giant component is quite sharp, which agree with the analysis using the theory of continuum percolation.

- Non-isolation probability serves as the upper bound of one-connectivity.

\[ \rho(P(C) = p) = \rho(P(\bar{I}) = p) + \delta, \delta > 0. \text{ As } p \rightarrow 1, \delta \rightarrow 0. \]

When the shadowing variance gets large, the difference \( \delta \) becomes smaller. This is because the increase of long links and the decrease of short links reduce the correlation between links. Thus the GRG approaches RG.
Discussion

- Theoretical performance of non-isolation probability matches simulated ones, except that when $\sigma$ is large ($\sigma=10$). This is because network suffers severe link asymmetry as shadowing variance increase. The theoretical calculation overestimates the global connectivity performance.
Discussion

- The increase of pass loss exponent $n$ shortens tx range; the increase in tx power increases the average tx range; impact of network size $|A|$ consists with the theoretical one.

- A different encoding scheme. SECDED (1:3) performs better than Manchester (1:2). Complicated encoding methods cost energy and memory space.

- Frame size change has a small influence.
Discussion

- Giant component
  - For practical use, one-connectivity may be too stringent.
  - Giant component probability is defined as the largest component size larger than N/2.
Future work

- Temporal dynamics of wireless connectivity
- The giant component needs further analytical study
- K-connectivity
Major References

The end!

Thanks