Connectivity in Wireless Sensor Networks

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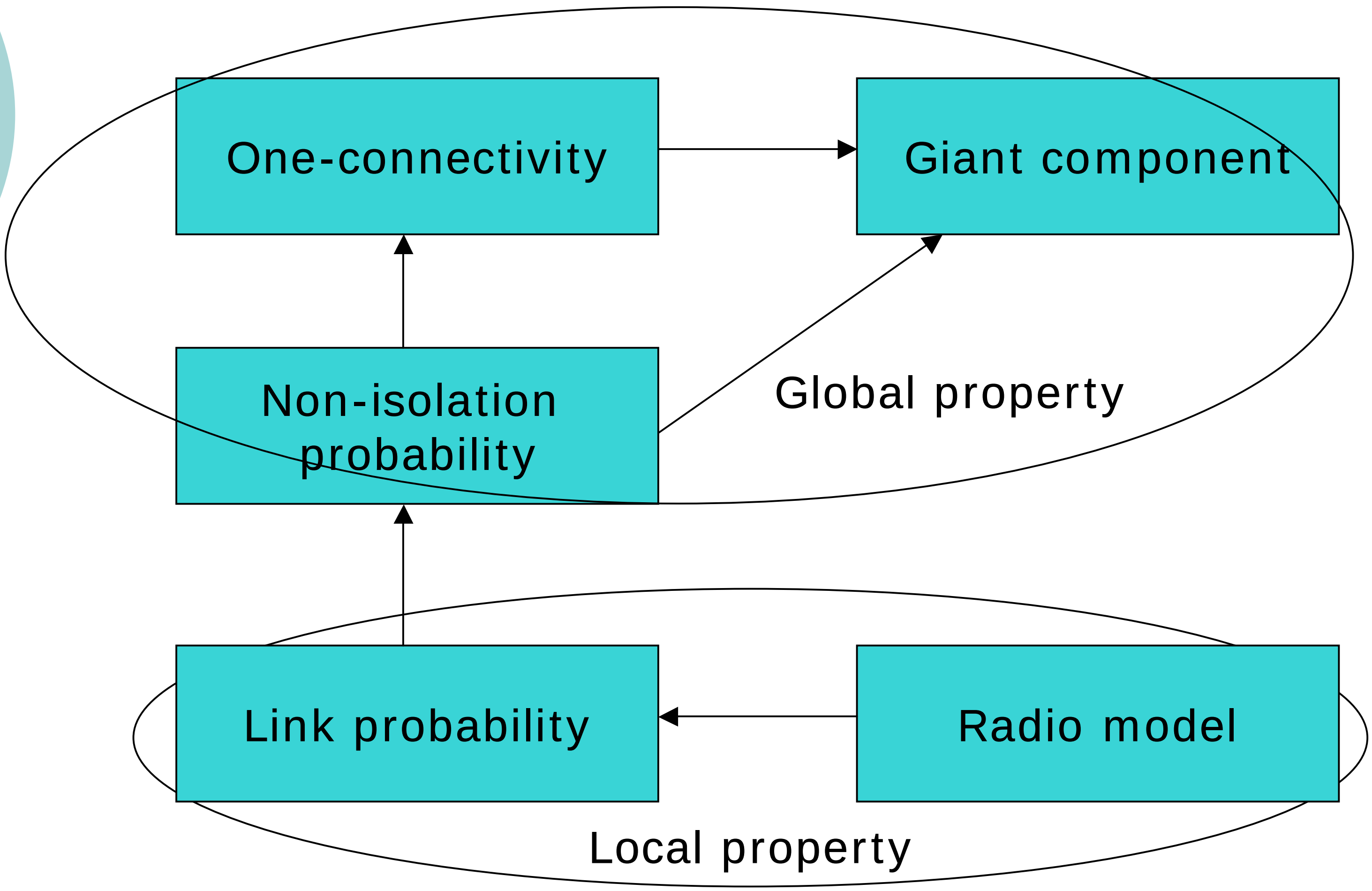
Outline

- Introduction and motivation
- Connectivity analysis
- Link probability analysis
- Simulation results
- Conclusion and future work

Introduction and motivation

- Connectivity is a fundamental property and provides design reference for upper-layer protocols.
- Boolean disk model is too simple, real wireless channel is dynamic, with path loss, fading and shadowing.
- Investigate connectivity under more realistic channel model, study how various parameters impact the quality of connectivity.

Main idea


One-connectivity
Giant component
Non-isolation probability
Global property
Link probability
Radio model
Local property

Connectivity analysis

- Node spatial distribution—homogenous Poisson point process

$N(A, \cdot)$ is the number of nodes in subarea A which follows Poisson distribution with mean $\lambda(A)$:

$$P(N(A) = k) = \frac{\lambda(A)^k}{k\,!} e^{-\lambda(A)} \quad , all\ A \in \mathcal{F}, \tag{1}$$

with an expected value $E(N) = \lambda(A) = \rho(A)\|A\|$, ρ and $\|A\|$ are node density and size of subarea A respectively.

If A_1, A_2, ... are disjoint sets then $N(A_1,)$, $N(A_2,)$, ... are independent random variables:

$$P(N(A_1) = k_1 \wedge N(A_2) = k_2 \wedge \ldots \wedge N(A_n) = k_n) = \prod_{i=1}^{n} P(N(A_i) = k_i). \tag{2}$$

Connectivity analysis

- One-connectivity probability *P(C)* vs. node non-isolation probability $P(\bar{I})$
  - Node non-isolation is the necessary but non-sufficient condition of one-connectivity, thus *P(C)* <= $P(\bar{I})$
  - Suppose the isolation of nodes to be independent events, we have:

$$
\begin{aligned}
P(\bar{I}) &= \sum_{k=0}^{\infty} P(\bar{I}|N=k) \cdot P(N=k) \\
&= \sum_{k=0}^{\infty} (1-P(I))^k \cdot \frac{\lambda^k}{k!} e^{-\lambda} \\
&= e^{-\lambda P(I)} \sum_{k=0}^{\infty} \frac{[(1-P(I))\lambda]^k}{k!} e^{-(1-P(I))\lambda} \\
&= e^{-\lambda P(I)}.
\end{aligned}
$$

Connectivity analysis

- P(I) is node isolation probability, D is node degree, D also follows Poisson distribution, we have:

$$P(I) = P(D = 0) = e^{-D_0}.$$

- D_0 can be calculated as

$$\begin{aligned} D_0 &= \rho \int_0^{2\pi} \int_0^{\infty} P(L|s)\ s\ ds\ d\phi \\ &= 2\pi\rho \int_0^{\infty} P(L|s)\ s\ ds. \end{aligned}$$

$$P(\bar{I}) = \exp(-\rho\|A\|\ \exp(-2\pi\rho\,\xi)), \quad \xi = \int_0^{\infty} P(L|s)\ s\ ds.$$

Link probability analysis

- Log-normal shadowing model

$$P_r(s) = P_t - PL(s_0) - 10\eta\log(\frac{s}{s_0}) + \mathcal{N}(0,\sigma),$$

- Incorporating anisotropic property

$$P_r(s) = P_t - (PL(s_0) + 10\eta\log(\frac{s}{s_0}))\cdot K_i + \mathcal{N}(0,\sigma),$$

$$K_i = \begin{cases} 1, & \text{if } i = 0; \\ K_{i-1} + rand \cdot DOI, & \text{if } 0 < i < 360 \wedge i \in N, \end{cases}$$

$$DOI = 0.01821.$$

Link probability analysis

- PRR as a function of BER, taking NCFSK, Manchester encoding for example:

$$\Psi(\gamma) = (1 - \beta_M(\gamma))^{8(2f-h)}, \qquad \beta_M = \frac{1}{2} e^{-\frac{\gamma}{2}}, \qquad \gamma = 10^{\gamma_{dB}/10}.$$

$$\Psi(\gamma) = (1 - \frac{1}{2} e^{-\frac{\gamma}{2}})^{8(2f-h)}.$$

- SNR can be obtained from the radio model

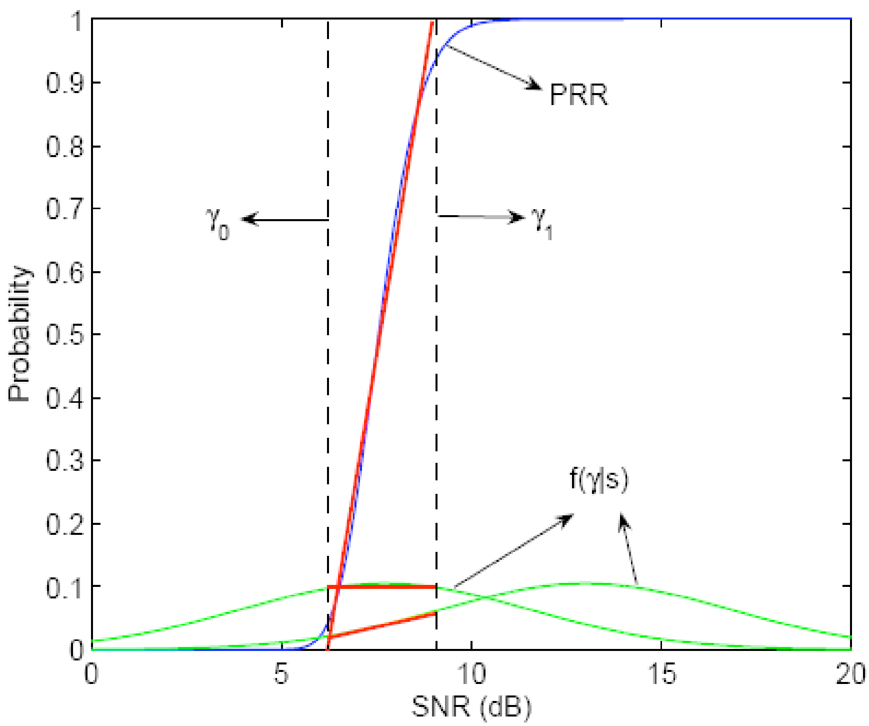
$$\gamma(s) = P_r(s) - P_n,$$

$$\mathcal{N}(\mu(s), \sigma) \qquad \mu(s) = P_t - (PL(s_0) + 10\eta \log(\frac{s}{s_0})) \cdot K_i - P_n.$$

- Finally,

$$\xi = \int_0^\infty P(L|s)\, s\, ds = \int_0^\infty \int_{-\infty}^\infty \Psi(\gamma) f(\gamma|s) \cdot s\, d\gamma\, ds.$$

Link probability analysis



$$\Psi(\gamma) = \begin{cases} 0, & \gamma \le \gamma_0; \\ k_\psi \gamma + b_\psi, & \gamma_0 < \gamma < \gamma_1; \\ 1, & \gamma > \gamma_1. \end{cases}$$

$$k_\psi = \frac{0.9 - 0.1}{\gamma_1 - \gamma_0}, b_\psi = \frac{0.1\gamma_1 - 0.9\gamma_0}{\gamma_1 - \gamma_0},$$

$$\gamma_0 = \Psi^{-1}(0.1), \gamma_1 = \Psi^{-1}(0.9).$$

$$f(\gamma|s) = k_f(s)\gamma + b_f(s), \quad \gamma \in [\gamma_0, \gamma_1],$$

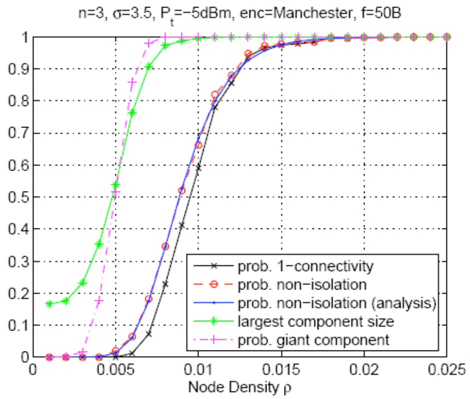
$$k_f(s) = \frac{f(\gamma_1|s) - f(\gamma_0|s)}{\gamma_1 - \gamma_0}, \quad b_f(s) = \frac{f(\gamma_0|s)\gamma_1 - f(\gamma_1|s)\gamma_0}{\gamma_1 - \gamma_0}$$

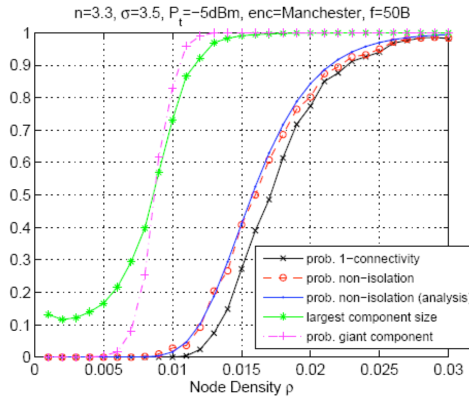
Link probability analysis

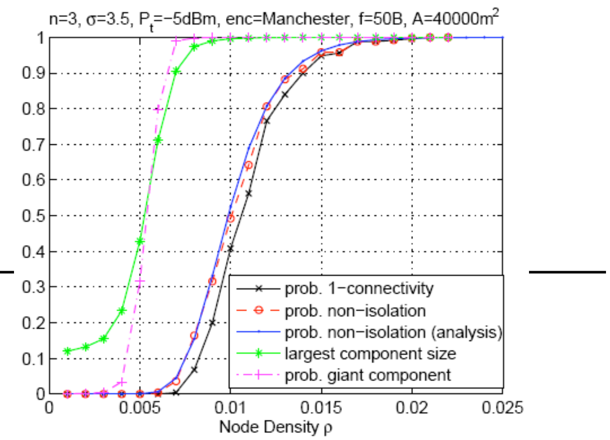
$$
\begin{aligned}
\xi &= \int_0^\infty \int_{-\infty}^\infty \Psi(\gamma) f(\gamma|s) \cdot s \, d\gamma \, ds \\
&\approx \int_0^\infty \int_{\gamma_0}^\infty \Psi(\gamma) f(\gamma|s) \cdot s \, d\gamma \, ds \\
&\approx \int_0^\infty (\int_{\gamma_0}^{\gamma_1} (k_\psi \gamma + b_\psi)(k_f(s)\gamma + b_f(s)) d\gamma + (1 - \Phi(\frac{\gamma_1 - \mu(s)}{\sigma}))) \, s \, ds \\
&\approx \int_0^\infty \int_{\gamma_0}^{\gamma_1} (k_\psi \gamma + b_\psi)(k_f(s)\gamma + b_f(s)) \cdot s \, d\gamma \, ds + \int_0^\infty (1 - \Phi(\frac{\gamma_1 - \mu(s)}{\sigma})) \, s \, ds \\
&\approx \int_0^{d_h} \int_{\gamma_0}^{\gamma_1} (k_\psi \gamma + b_\psi)(k_f(s)\gamma + b_f(s)) \cdot s \, d\gamma \, ds + \int_0^{d_h} (1 - \Phi(\frac{\gamma_1 - \mu(s)}{\sigma})) \, s \, ds,
\end{aligned}
$$

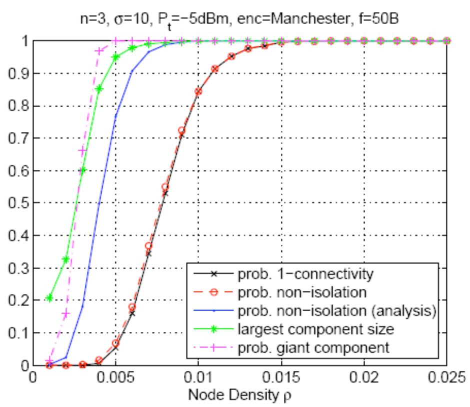
Simulation settings

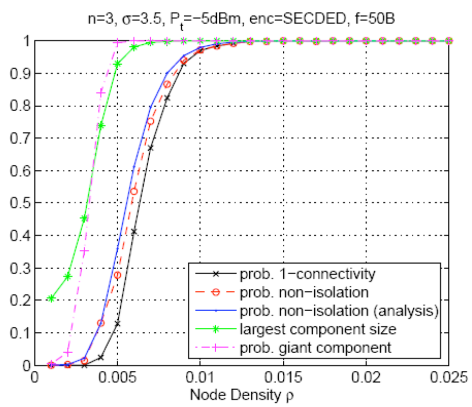
- Select a subarea, eliminating border effect
- Baseline: $n=3, \sigma=3.5$, $Pt=-5dBm$, encoding method=Manchester, frame size=50 bytes, $|A|=20000m^2$ (modulation=NCFSK)
- Comparison: theoretical-simulation results of non-isolation prob.; one-connectivity vs. non-isolation prob.; giant component vs. one-connectivity; impact of different parameters.

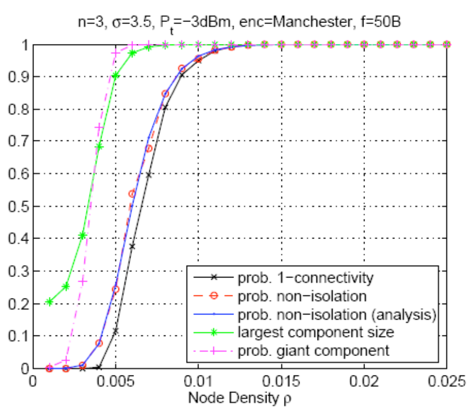

n=3, σ=3.5, P$_t$=−5dBm, enc=Manchester, f=50B
Node Density ρ
prob. 1−connectivity
prob. non−isolation
prob. non−isolation (analysis)
largest component size
prob. giant component
(a)

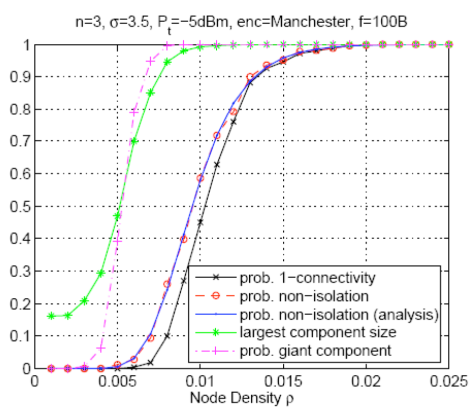

n=3.3, σ=3.5, P$_t$=−5dBm, enc=Manchester, f=50B
Node Density ρ
prob. 1−connectivity
prob. non−isolation
prob. non−isolation (analysis)
largest component size
prob. giant component
(b)


n=3, σ=3.5, P$_t$=−5dBm, enc=Manchester, f=50B, A=40000m^2
Node Density ρ
prob. 1−connectivity
prob. non−isolation
prob. non−isolation (analysis)
largest component size
prob. giant component


n=3, σ=10, P$_t$=−5dBm, enc=Manchester, f=50B
Node Density ρ
prob. 1−connectivity
prob. non−isolation
prob. non−isolation (analysis)
largest component size
prob. giant component
(c)


n=3, σ=3.5, P$_t$=−5dBm, enc=SECDED, f=50B
Node Density ρ
prob. 1−connectivity
prob. non−isolation
prob. non−isolation (analysis)
largest component size
prob. giant component
(d)


n=3, σ=3.5, P$_t$=−3dBm, enc=Manchester, f=50B
Node Density ρ
prob. 1−connectivity
prob. non−isolation
prob. non−isolation (analysis)
largest component size
prob. giant component
(e)


n=3, σ=3.5, P$_t$=−5dBm, enc=Manchester, f=100B
Node Density ρ
prob. 1−connectivity
prob. non−isolation
prob. non−isolation (analysis)
largest component size
prob. giant component
(f)

Discussion

- As node density increases, the transition from low connectivity and the appearance of giant component is quite sharp, which agree with the analysis using the theory of continuum percolation.
- Non-isolation probability serves as the upper bound of one-connectivity.
  $\rho(P(C) = p) = \rho(P(\bar{I}) = p) + \delta, \delta > 0.$ As $p \to 1, \delta \to 0.$

  When the shadowing variance gets large, the difference δ becomes smaller. This is because the increase of long links and the decrease of short links reduce the correlation between links. Thus the GRG approaches RG.

Discussion

- Theoretical performance of non-isolation probability matches simulated ones, except that when σ is large (σ=10). This is because network suffers severe link asymmetry as shadowing variance increase. The theoretical calculation overestimates the global connectivity performance.

Discussion

- The increase of pass loss exponent *n* shortens tx range; the increase in tx power increases the average tx range; impact of network size |A| consists with the theoretical one.
- A different encoding scheme. SECDED (1:3) performs better than Manchester (1:2). Complicated encoding methods cost energy and memory space.
- Frame size change has a small influence

Discussion

- Giant component
  - For practical use, one-connectivity may be too stringent.
  - Giant component probability is defined as the largest component size larger than N/2.

Future work

- Temporal dynamics of wireless connectivity
- The giant component needs further analytical study
- K-connectivity

Major References

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- Miorandi, D.; Altman, E. Coverage and connectivity of ad hoc networks presence of channel randomness. In *Proc. IEEE INFOCOM* 2005, 491–502.
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The end!

Thanks