Highly Accurate Frequency Estimation for FNET

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***Abstract*—Frequency disturbances is one of the most important indicator that reflects the state of an electrical power system. Making accurate frequency measurement from low voltage distribution systems through the wide deployment of Frequency Disturbance Recorders (FDRs) has been the major innovation for the Frequency monitoring network (FNET). Currently, the frequency calculation algorithm based on the phasor angles (FPA) of the measured voltage signal in the FDR, has achieved high accuracy. However, this algorithm is very sensitive to noise which is inevitable in the signal sampling process. As a result, the frequency estimation accuracy will be degraded if the measured signal is not clean enough. Therefore, to achieve even more accurate and robust frequency estimation from the measured voltage signal, a novel algorithm is proposed in this paper consisting of two stages of operations. The first stage applies a band pass filter that eliminates the irrelevant harmonics; while the second stage removes the noise within the pass band and estimates the frequency fluctuation simultaneously based on the least squares nonlinear curve fitting. The experiments based on synthetic data and real data validates the effectiveness of the proposed method for improving the accuracy of frequency estimation.**

***Index Terms*--Frequency Estimation, Least Square, Nonlinear Curve Fitting, Frequency Disturbance Recorder, Frequency Monitoring Network, Power Grid.**

I. INTRODUCTION

Frequency information is very useful in many areas of power system analysis, operation and control. During the past years, many advances have been made in the areas of accurate and efficient frequency information retrieval. During the 1980s, work first began on closely synchronized measurements that would allow direct measurement of voltage phase angle at the transmission level. As a result, Phasor Measurement Units (PMU) [1,2] have been gradually installed in substations and measure phasor at high voltage levels. As a member of the PMU family, the Frequency Disturbance Recorder (FDR) was developed at Virginia Tech in 2003. The FDR collects instantaneous voltage phasor and frequency measurements at low voltage distribution level using the ordinary 120V wall outlets and then transmits the measured frequency data remotely via the Internet. Based on these low-cost FDRs, a US-wide Frequency Monitoring Network (FNET) has been implemented [3,4,5]; it serves the entire North American power grid through advanced situational awareness techniques, such as real-time event alert, accurate event location estimation, animated event visualization, and post event analysis [6].

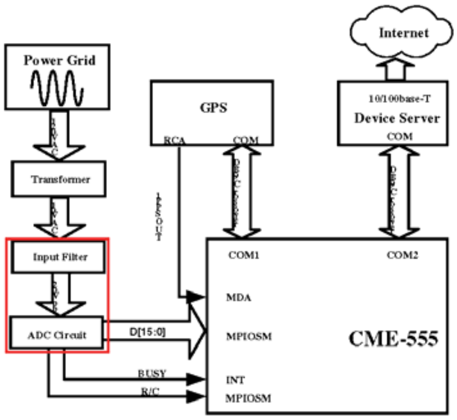

Figure 1. Architecture of FDR unit [7]

The FDR architecture is demonstrated in figure 1. Every FDR is a single-phase phasor measurement unit (PMU) in the sense that it measures the voltage phase angle, amplitude, and estimate frequency from a single-phase voltage source.

During the past several decades, many kinds of frequency estimation algorithms have been proposed. The Zero-crossing method might be the most trivial with minimized hardware configuration and low computational cost. However, the accuracy of the frequency estimation is heavily affected by the noise in signal. Although the technique for leakage reduction is easy to be implemented, its accuracy suffers a lot from non-fundamental components. As a result, an extra zero crossing detector is necessary since this technique works only for a sinusoidal waveform [7]. The Kalman filter is good for denoising during measurement, but the probability density function of the state vector has to be estimated empirically. The state-of-the-art technique for frequency estimation is the phasor angle analysis technique, already deployed in the commercialized FDR device. However, the algorithm is still sensitive to noise [7,8].

In order to make the frequency estimation more reliable and robust, this paper presents a novel solution that consists of

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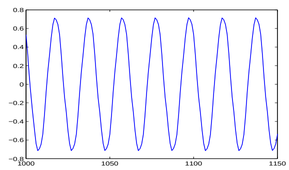
two steps: first, a band-pass filter is specifically designed to filter the irrelevant harmonics in the voltage signal; second, to remove the remaining noise in the pass band, we use the least square sinusoidal curve fitting to resemble every short period voltage signal, which should be very close to the sinusoidal wave. In this way, the second step removes the remaining noise in pass band and estimates the frequency of every short period simultaneously. The proposed two-step operation is able to greatly improve the accuracy of frequency estimation based on the measured noisy voltage signal.

II. BAND PASS FILTER DESIGN

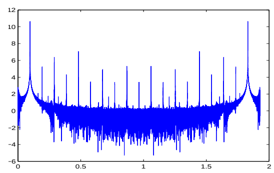
Currently the FDR devices integrate a native low-pass filter, as shown in figure 1. Given the fact that the frequency of a real power grid, such as the Eastern Interconnection (EI) and the Western Electricity Coordinating Council system (WECC), the possible frequency falls into the 59-61Hz interval unless a blackout happens resulting in a collapse of the whole system, it should be better if we take the band pass filter instead of the low pass filter. Therefore, a normal band pass filter implemented by the finite impulse response (FIR) filter considering feasibility and efficiency is first designed.

A. Charactertistics of the Voltage Signal

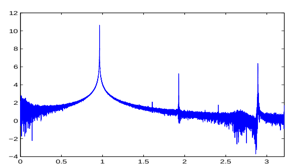
The raw voltage signal collected by FDR from the power grid is obviously quite noisy, as shown in figure 2a. From the frequency spectrum plot shown in figure 2b, where the x-axis indicates frequency in Hz and the y-axis indicates amplitude in dB, we observe that harmonics around the 60Hz signal and Gaussian white noise are dominant and non-negligible. Besides the peak frequency around 60 Hz, there also exist impulses around 120Hz and 180Hz along with the irregular fluctuations and ripples in the background, as shown in figure 2c.



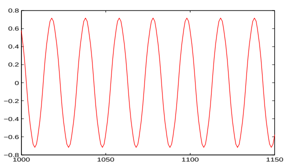
(a) Voltage signal in time domain



(b) Signal frequency spectrum.



(c) Frequency spectrum 0-200hz



(d) Band pass denoised Signal.

Figure 2. Voltage signal measured in the power grid.

The mathematical expression for the signal in frequency domain can be denoted as:

$$x_f(k) = s_f(k)+\sum h_i(k)+n_f(k) \qquad (1)$$

where $s_f(k)$ is the sinusoidal signal with frequency around 60Hz, $h_i(k)$ is the harmonics impulses, and $n_f(k)$ is the Gaussian white noise with zero mean and variance σ_n^2. Note that the voltage signal and its corresponding frequency spectrum are all discrete.

B. Filter Design

Since the interval between the sinusoidal signal and its nearest harmonic impulse is approximately 60Hz, and the wider the pass band, the lower order the pass band, the width of the pass band can just be merely less than 2×60Hz. On the other hand, considering the frequency of the white noise spreads throughout the whole frequency spectrum, the pass band should be narrow enough to only cover the dynamic range of the voltage signal frequency range, i.e., an interval of 59-61Hz. Trade-off should be made, and we choose a pass band between 55-65Hz. A Kaiser window is used to represent the pass band, since Kaiser windows are nearly optimal and allow controlled trade-offs between the main lobe widths and side lobe amplitudes [9]. Meanwhile the transition bandwidth is 5Hz on both higher and lower sides of the pass band; the attenuation of the stop-band is set to be 40dB.

Figure 3 shows the voltage signal in top subfigure applied by the filter whose impulse response is shown in the middle subfigure. Comparing the top and bottom subfigures, it is obvious that the harmonics and the majority of the white noise are removed. It should be pointed out that due to the frequency spectral leakage in FFT, the signal-to-noise ratio (SNR) of the signal does not match the performance of the band-pass filter. This leakage is the consequence when the sampling rate is not integer times of the signal frequency. Unfortunately the utility frequency is not always 60Hz and varying all the time within an extreme tiny range, which is already enough to cause the leakage, but this can be solved by the re-sampling technique.

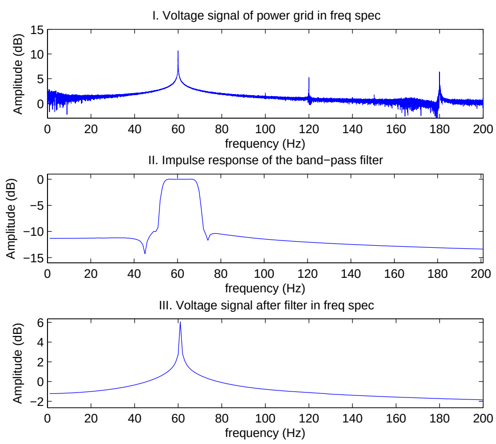


Figure 3. Frequency spectrum of raw signal and filtered signal.

III. FREQUENCY ESTIMATION WITH DENOISED SIGNAL

After the first step of denoising, most of the harmonics outside of the pass band are removed. However, part of the white noise still resides within the pass band, affecting the accuracy of frequency estimation using the existing FPA algorithm.

Before we describe the proposed method, the FPA algorithm is briefly reviewed here. Given a voltage signal, denoted as *S*, a moving window ζ with fixed length is applied on *S*. By moving the window ζ with a fixed step ω, whose value is less than the length of the window, we can crop a sequence of short period signals, denoted as s_1, s_2, s_3, ... With each signal fragment s_i, the FPA algorithm is utilized to estimate the value of frequency at the current time period i. The FPA algorithm is a recursive algorithm that consists of two major steps: first, a rough frequency estimation in the signal fragment s_i is computed using a second order least square approximation on the phasor angles; then, a resampling based on the rough frequency estimation is carried out, followed by another second order least square approximation to obtain the final estimation. This FPA algorithm is able to achieve high accuracy for frequency estimation with clean input signal. However, even with the denoised signal from Sec. II, it still produces oscillations around the true frequency value due to noise within the pass band.

Since the frequency calculation based on phasor angle is not robust enough to noise, we propose to use the least square nonlinear curve fitting [10] technique for frequency estimation. The general idea is that instead of calculating the frequency value from the signal fragment s_i directly, we can reproduce a pure sinusoidal fragment wave s^*_i to approximate the fragment s_i, with the squared error between the s^*_i and the s_i as small as possible. Expressed in mathematical formula, the solution is to find the function parameters (e.g. frequency, amplitude) ϕ that solve the problem:

$$\min_{\phi}\|F(\phi, kdata) - s_i\|^2 = \min_{\phi}\sum_k (F(\phi, k) - s_i(k))^2 \quad (2)$$

where F is a sinusoidal function with no harmonics, thus can be defined just by four parameters: amplitude, frequency, phase, and base level $(\alpha,\theta,\varphi,\eta)$. This optimization problem can be solved by resorting to the "trust-region-reflective" algorithm [10,11], and the initial Levenberg-Marquardt parameter λ is set as 0.01. Note that this algorithm is sensitive to the initial setting of the parameters $(\alpha,\theta,\varphi,\eta)$. To enable the algorithm to converge faster, we can roughly estimate the amplitude α, frequency θ and base level η by detecting the cross-zero points in the signal fragment s_i, and set phase φ as a random value between 0-π. If the optimized parameters are not reasonable, change the value of phase φ and run it another time. Repeated trials will be conducted until the solution converges to real optimal parameters.

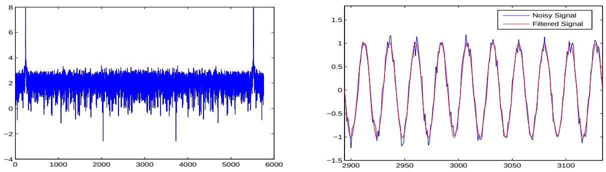
This frequency estimation strategy calculates frequency from a completely new perspective where we obtain a pure sinusoidal signal fragment s^*_i that resembles s_i based on the four optimal parameters. This strategy brings two immediate advantages. First, since the error between s^*_i and s_i is contributed mainly by the remaining noise in the pass band, by producing this pure sinusoidal fragment, we can obtain a further denoised signal S^* by putting all the new fragments s^*_i together. Second, with these optimized parameters, we also can give the estimation of frequency during each period based on the band-pass filtered signal S. Hereinafter, we refer to this algorithm as frequency estimation based on curve fitting (FCF). Although the FCF algorithm is slower than FPA if taking the same size window and the moving step ω, we can set ω 10 times larger, and re-input the produced pure sinusoidal s^*_i into the FPA algorithm to obtain the same resolution of frequency estimation as FPA. Finally, we can achieve highly accurate frequency estimation with just a bit more computational cost.

IV. Experiment and Results

To evaluate the proposed frequency estimation method, we conduct several experiments based on both synthetic data and real data.

A. Evaluation of Band-Pass Filter

In order to evaluate the performance of the band pass filter, we use the signal to noise ratio (SNR) as a metric. Since the real value of frequency in the real voltage signal sampled from power grid is unknown (the frequency calculated by any algorithm would only be an approximation of the actual value, no matter how accurate it claims), we synthesize a pure sinusoidal signal with white noise added in MATLAB. The SNR of the noisy signal is set as 15dB. The frequency spectrum is shown in figure 4a. After the filtering, SNR is calculated with the ground truth. From figure 4b, we can see the filter removed a great portion of the noise, and the SNR is improved to 26.52dB, indicating the effectiveness of the band pass filter for pre-denoising in frequency estimation.



(a) Noisy signal frequency spectrum. (b) Noisy signal and filtered signal.

Figure 4. Evaluation of band pass filtering.

B. Evaluatation with Synthetic Voltage Signal Data

To evaluate the FCF algorithm performance for frequency estimation, we synthesize two benchmark voltage signals: signal S_1 is a sinusoidal wave of stable frequency as 59.98Hz with white noise added, the other signal S_2 is a sinusoidal wave of unstable frequency varying around 58.98hz with also white noise added. The SNR of both noisy signals S_1 and S_2 are 20dB. As for the processing details, in all the tests of this section we set the length of each signal fragment s_i as about 8 cycles and set the window's moving step ω as 1 and 10 for FPA and FCF, respectively. The mean squared error (MSE) between the estimated frequency fluctuation and the ground truth is used for quantitative evaluation.

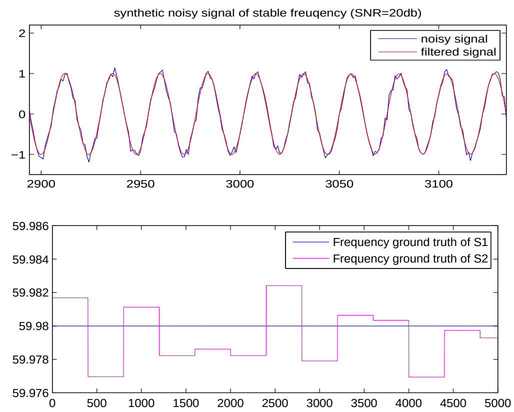


Figure 5. Synthetic voltage signal S_1, and the frequency of S_1 and S_2

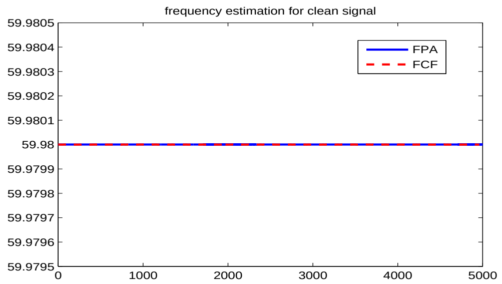


Figure 6. Frequency estimation with clean signal.

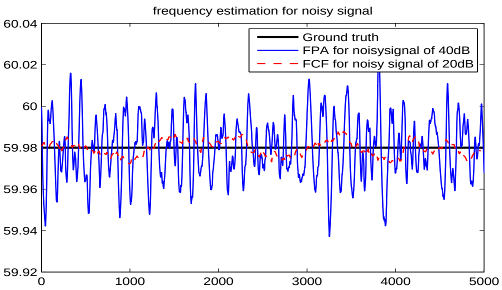


Figure 7. Frequency estimation with noisy signal.

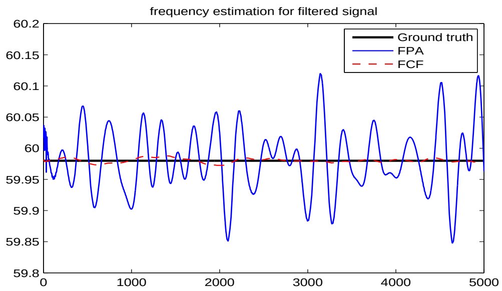


Figure 8. Frequency estimation with filtered signal.

TABLE I. TIME COST AND MSE OF TWO ALGS WITH TEST ON S_1.

S_1	*CleanSig*	*NoisySig 20dB*	*NoisySig 40dB*	*FilteredSig*
Time$_{_FPA}$	33.9 s		31.1 s	31.1s
Time$_{_FCF}$	101.8 s	93.19 s	87.9 s	100.7 s
MSE$_{_FPA}$	4.15e-15		2.24e-4	2.80e-3
MSE$_{_FCF}$	3.75e-22	1.36e-5	8.14e-8	1.22e-5

First of all, the FPA and FCF algorithms are both applied on clean signal S_1 without any noise. Figure 6 shows that both FPA and FCF can give highly accurate frequency estimation around the ground truth, i.e., stable frequency of 59.98hz with almost no oscillation, for very clean signal. However, the MSE shows that FCF achieves more accurate frequency estimation - the MSE of FCF can be 10e7 times smaller than that of FPA.

Second, the FPA and FCF algorithms are applied on noisy signal S_1, whose SNR is 20dB. The FPA cannot give stable

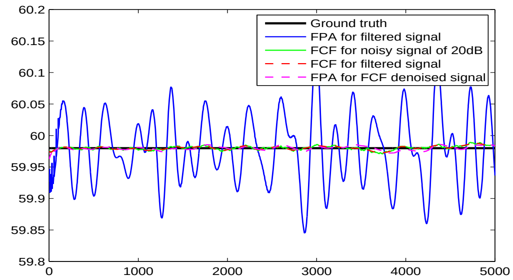


Figure 9. Input the FCF denoised signal into FPA for higher resolution.

frequency estimation with white noise added. We increase the signal SNR to 40dB for FPA, while FCF is still used to estimate the S_1 with SNR of 20dB. The result is shown in figure 7. We observe that even with a much more noisy signal, the FCF can still give much better frequency estimation than FPA. The MSE of estimation on S_1 by FCF is 1.36e-5, which is even smaller than the MSE (2.24e-4) of estimation on the less noisy S_1 (40dB) by FPA.

Third, the FPA and FCF algorithms are both applied on the band-pass filtered signal $S^b{}_1$, whose SNR is improved by the filter to 27.6dB. The frequency estimation is shown in figure 8. It is obvious that the FCF again gives much better estimation. The MSE of estimation by FCF is 230 times smaller than that given by FPA. Furthermore, the FCF is very robust to noise. From table 1, we can find the FCF gives similar accurate frequency estimation for both signals with and without band pass filtering, where the MSE are 1.22e-5 and 1.36e-5, respectively.

It is obviously that the FCF algorithm can achieve much higher frequency estimation than FPA, though the FCF is 3 times slower than the FPA algorithm. In addition, FCF moves the window at every 10 samples while FPA moves at every 1 sample, thus the resolution of the estimated frequency of FCF is 10 times lower. To solve this problem, we can use the FCF as a second step of denoising, since it can reproduce a pure sinusoidal fragments s^*_i based on these optimized parameters in each window. The merged s^*_is can be formed as a complete perfectly denoised signal S^*. Then, we input S^* into FPA algorithm again for high resolution frequency estimation. This result is shown in figure 9, from which we can see the FPA algorithm performs similar to FCF with the optimal filtered signal, and the MSE of estimation is 1.33e-5.

We have also done the same experiment with the synthetic signal S_2, whose frequency fluctuates with time as shown in the bottom of figure 5. The experimental results are shown in figure 10 and table 2, from which it is easy to find that though the FCF costs more computation (around 3-4 times), the accuracy of the frequency estimation it improved is at least more than 10 times. In addition, the FCF algorithm is very robust to the white noise - it can obtain almost similar accurate estimation even without the band-pass filtering. As for the resolution of the frequency estimation, it can be solved by re-input the FCF denoised signal into FPA, to take advantage of the estimation accuracy, resolution and robustness at acceptable speed.

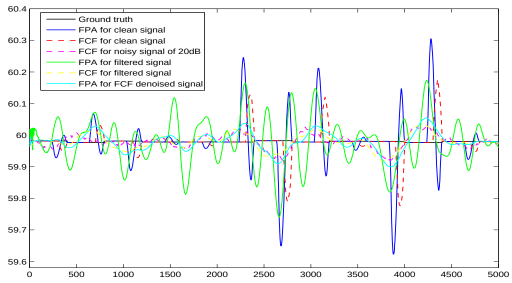


Figure 10. Frequency estimation with frequency unstable signal S2.

TABLE II. TIME COST AND MSE OF TWO ALGS WITH TEST ON S_S.

S_2	*CleanSig*	*NoisySig 20dB*	*NoisySig 40dB*	*FilteredSig*
Time _FPA	29.61 s		27.03 s	27.75 s
Time _FCF	101.05 s	103.95s		104.11 s
MSE _FPA	4.72e-3		4.73e-3	5.41e-3
MSE _FCF	1.60e-3	5.35e-4		5.32e-4

C. Evaluatation with Real Voltage Signal Data

The real data is sampled from the wall outlet by using the FDR directly, denoted as S_r. First of all, we perform the band pass on the raw voltage signal. The original and filtered signal are both shown in figure 11, from which we can see the filtered signal is more clear and close to pure sinusoidal wave.

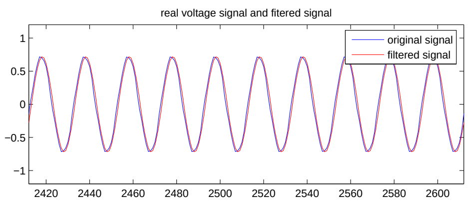


Figure 11. real voltage signal and band pass filtered signal.

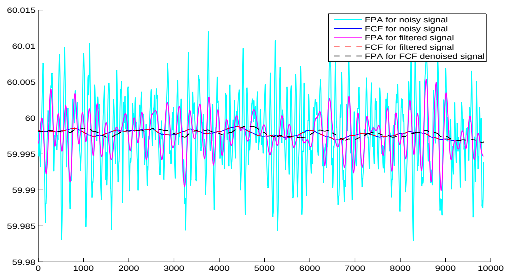


Figure 12. Frequency estimation with real voltage signal.

Then, both the FPA and the FCF algorithms are utilized to estimate the frequency fluctuation of the real voltage signal. We applied both algorithms on the original noisy signal and the band pass filtered signal. The result is shown in figure 12. The ground truth of the frequency is unknown, but we know the electrical voltage is pretty stable during the time when the data is sampled. Therefore, a stable frequency level is an appropriate assumption. Under this assumption, we can see that the FCF algorithm performs much better than the FPA algorithm. The frequency estimation given by FCF is very stable compared to that given by the FPA algorithm. Though the running time is again about 3 times slower for FCF compared to FPA, the improved accuracy should be more than 50 times. In addition, we can see with original noisy signal and band pass filtered signal, the FCF is able to give almost the same frequency estimation, indicating its robustness to noise in estimation. Finally, the resolution problem can be solved by re-input the FCF denoised signal into FPA algorithm, and obtain frequency estimation with both high accuracy and high resolution.

V. CONCLUSION

In this paper, we proposed a two-step strategy for highly accurate frequency estimation from measured voltage signal. While most of the previous methods focus on the calculation of the frequency information from the denoised signal directly, we took a reserve approach. With each signal fragment, we could find the optimal parameters to construct the sinusoid with the least squared nonlinear curving fitting algorithm. With these optimal sinusoidal parameters, we were able to perform denoising and frequency estimation simultaneously. Experiments with both synthetic data and real data showed that the proposed FCF algorithm achieved much higher accuracy on frequency estimation and much stronger robustness to noise in signal than the state-of-the-art technique.

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