Coverage Estimation for Crowded Targets in Visual Sensor Networks

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Coverage estimation is one of the fundamental problems in sensor networks. Coverage estimation in visual sensor networks (VSNs) is more challenging than in conventional 1-D (omnidirectional) scalar sensor networks (SSNs) because of the directional sensing nature of cameras and the existence of visual occlusion in crowded environment. This paper represents a first attempt toward a closed-form solution for the visual coverage estimation problem in the presence of occlusions. We investigate into a new target detection model, referred to as the “certainty-based target detection” as compared to the traditional “uncertainty-based target detection” to facilitate the formulation of the visual coverage problem. We then derive the closed-form solution for the estimation of the visual coverage probability based on this new target detection model that takes visual occlusions into account. According to the coverage estimation model, we further propose an estimate of the minimum sensor density that suffices to ensure a visual K-coverage in a crowded sensing field. Simulation is conducted which shows extreme consistency with results from theoretical formulation, especially when the boundary effect is considered. Thus the closed-form solution for visual coverage estimation is effective to be applied to real scenarios such as efficient sensor deployment and optimal sleep scheduling.

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General Terms: Visual sensor network, coverage estimation
Additional Key Words and Phrases: directional coverage, visual occlusions

1. INTRODUCTION

Vision is perhaps the most powerful of the human senses. Many multi-camera systems have been developed for different applications ranging from security monitoring to surveillance in the last few decades. In these applications, expensive and high-resolution cameras are usually deployed into large buildings (i.e., malls and airports) and open areas (i.e., parking lots and public parks) to capture the events in a controlled sensing field. In general, the position and orientation of cameras are predetermined and well-ordered to optimize the placement of cameras. However, in a hostile and dangerous environment (e.g., battlefield), it is not possible or feasible to deploy the cameras with accurate position and orientation. Therefore, camera
nodes might be randomly deployed into the sensing field from a moving platform (e.g., airplane or vehicle) in order to monitor the environment [Hynes et al. 2004].

Due to their high cost and significant power consumption, random deployment of camera nodes in a sensing field was not practical in the past. However, the production of small-size, low-power and affordable visual sensor platforms with imaging, on-board processing, and wireless communication capabilities has come to reality with recent advances in imaging, networking, embedded computing and circuit design technologies [Rinner and Wolf 2008]. Representative platforms which cost less than $300 include the Stanford MeshEye system [Hengstler et al. 2007], UC Berkeley’s CITRIC camera node [Chen et al. 2008], the WiCa (wireless camera) node from NXP research and Philips research [Abbo and Kleihorst 2002], UCLA’s Cyclops [Rahimi et al. 2004], CMU’s CMUCam [Rowe et al. 2007], and the most recent DSPcam [Kandhalu et al. 2009]. Deployment of a large number of such platforms to cover a large surveillance area forms a so-called visual sensor network (VSN) that is capable of solving computer vision problems through distributed sensing and collaborative in-network processing.

One of the fundamental issues in VSN deployment is coverage estimation. Due to random deployment of sensor nodes, their positions may not be predetermined. Additionally, due to the large amount of sensors deployed, it is impractical to manipulate node locations after deployment in order to reach a desired visual coverage [Akyildiz et al. 2002]. Therefore, to have proper sensor coverage in the sensing field, some sensor related parameters, such as sensor density, sensing range, etc., should be decided based on the estimated sensor coverage probability before deployment.

Traditionally, coverage probability has been evaluated based on the total number of sensor nodes that captures an arbitrary target within their sensing range. If every target in the sensing field is captured by at least K sensor nodes, it is called a K-covered sensor network [Li and Kao 2010]. Coverage estimation in VSNs is more challenging than in conventional omnidirectional scalar sensor networks (SSNs) because of two unique features of cameras [Qian and Qi 2008]. First of all, visual sensors provide limited and directional field of views (FOVs). Secondly, because of the crowded targets in the sensing field, “visual occlusions” among targets cannot be avoided. Therefore, coverage estimation in a crowded VSN depends not only on the sensor density and deployment but also on the target density and distribution.

In this paper, we focus on the formulation of a closed-form solution for the visual K-coverage estimation in VSNs with the presence of visual occlusion among crowded targets where a large number of visual sensor nodes has already been deployed. Having a closed-form solution for the coverage estimation problem facilitates many application deployments in VSNs. For example, efficient deployment of the sensor nodes can be achieved with minimum sensor density. Additionally, effective algorithms can be designed to yield optimal sensor sleep scheduling for energy saving purpose [Cai et al. 2009].

To formulate the visual coverage probability in the crowded environment, we first need to investigate into the target detection model. Traditionally, targets are detected based on the identification of intersections of the back-projected 2D cones of the targets. However, the existence of visual occlusion among targets would generate many empty intersections (false alarms) and partial appearance of
targets which makes the derivation of a closed-form solution for visual coverage estimation extremely difficult. In this paper, instead of resolving the uncertainty about target existence at the intersections, we model the target detection algorithm based on distributed camera nodes integrating the target non-existence information within the camera’s field of view at each sensor node. According to this target detection model, we then construct a closed-form solution to estimate the visual coverage probability that deals with the directional sensing nature of cameras and the visual occlusions among crowded targets. Based on the closed-form solution of the coverage estimation, we further propose an estimate for the minimum sensor density that suffices to ensure a visual K-coverage in a crowded sensing field.

The main contributions of this paper are two-fold:

(1) We derive a closed-form solution for visual coverage estimation in a randomly deployed VSN by adapting the non-existence information based target detection model into formulation. Therefore, the sensor related parameters (e.g., sensor density, sensing range, etc.) can be decided before deployment in order to have proper visual coverage in the sensing field. This facilitates many application deployments such as efficient sensor deployment and sensor scheduling in VSNs.

(2) In a crowded environment, the visual coverage probability depends not only on the sensor density and deployment but also on the target density and distribution. Our closed-form solution considers both the directional sensing nature of cameras and the visual occlusions among targets to estimate the visual coverage in VSNs. Thus, we have more accurate and more realistic visual coverage estimation in a crowded VSN.

The remainder of the paper is organized as follows: Section 2 describes the background and related works. Section 3 presents the target detection model. In Section 4, we provide the closed-form solution for visual coverage when occlusion is not taken into consideration. And in Section 5, we consider the more complex problem with occlusion taken into account. To show how the proposed target detection model enables the closed-form solution for visual coverage estimation, we present a detailed comparison between the proposed and traditional target detection models for visual coverage estimation in Section 6. Section 7 investigates into the complicated boundary effect for more accurate visual coverage estimation. Based on the closed-form solution of visual coverage estimation, Section 8 formulates the minimum sensor density estimation problem as an application example. Section 9 presents the experimental results to validate the theoretical derivation of visual coverage estimation and to show the effects of various parameters on the minimum sensor density. Finally, we conclude and discuss future works in Section 10.

2. BACKGROUND AND RELATED WORKS

In literature, there exist many works related to coverage estimation in scalar sensor networks (SSNs) where the sensing devices are normally 1-D omnidirectional (e.g., acoustic or seismic sensor). In order to effectively cover the given sensing region, various criteria have been considered, including quality of surveillance [Gu and Mohapatra 2004], maximal or minimal exposure of a path [Veltri et al. 2003], area coverage [Ahmed et al. 2005], etc. In SSNs, the area coverage of a sensor node is
modeled by a simple omnidirectional sensing model as a circular disk whose radius, \( \rho \), is the sensing range of the sensor node [Huang and Tseng 2003].

In [Meguerdichian et al. 2001], the coverage problem in SSNs was defined from several point of views including deterministic, statistical, worst and best case to determine the lower and higher observability in sensing field by combining computational geometry and graph theoretic techniques. [Xing et al. 2005] presented a design of coverage configuration protocol that can dynamically configure a network to achieve guaranteed degrees of coverage and connectivity and provide a geometric analysis of the relationship between coverage and connectivity. [Kumar et al. 2005] introduced the K-barrier coverage for a belt-shape region and established the optimal deployment pattern to achieve it. In [Wan and Yi 2006], the effect of the sensing radius or the total number of deployed sensor nodes on the probability of the K-coverage was studied for randomly deployed scalar sensor nodes and the boundary effect was taken into account. In [Brass 2007], the coverage estimation problem was analyzed with the Boolean sensing model for either mobile or stationary sensors and targets, under random or optimal placement. [Yen et al. 2006] proposed a mathematical expression to predict the coverage rate for an expected area in a wireless sensor network that can be K-covered to determine the related sensing parameters. [Wang et al. 2007] considered the coverage problem from the perspective of target localization to estimate the minimum sensor density to keep the target localization error within an acceptable bound.

Different from the scalar sensors, the sensing region of a camera, also referred to as the field of view (FOV), is limited and directional which is less than 180° in general. Therefore, existing works related to SSNs cannot be directly applied to visual sensor networks. In [Ai and Abouzeid 2006], the directional sensor coverage problem was investigated by utilizing linear programming to maximize the sensor coverage with minimum number of sensors. An energy-efficient target-oriented sleep scheduling algorithm was presented in [Cai et al. 2009] to extend the lifetime of directional sensor networks. [Liu et al. 2008] proposed directional and effective sensing models to capture the frontal view of the human face for orientation detection. Meanwhile, other research efforts [Yang et al. 2004; Isler and Bajcsy 2006] applied directional coverage analysis to minimize the sensor density to reach the accurate estimation for target localization and occupancy reasoning, respectively.

Since the coverage issue in VSNs is also related to the orientations of cameras, the problem of selecting a minimum number of sensors has been investigated based on automatic control of visual sensors by reorienting the deployed cameras to provide best possible coverage on a given area or targets. [Munishwar and Abu-Ghazaleh 2010] presented a novel centralized force-based approach to compute near optimal solutions using integer linear programming in a large-scale PTZ (pan, tilt, and zoom) camera network. [Fusco and Gupta 2009] designed a simple greedy algorithm that delivers a solution for selecting and orienting visual sensors that K-covers at least half of the target points using at most \( M \log(k|C|) \) sensors, where \(|C|\) is the maximum number of target points covered by a sensor and \( M \) is the minimum number of sensors required to K-cover all the given points. For more detailed survey, readers may refer to [Guvensan and Yavuz 2011] where the existing coverage optimization and enhancement solutions for directional sensor networks

were classified into four categories as target-based coverage enhancement, area-based coverage enhancement, coverage enhancement with guaranteed connectivity, and network lifetime prolonging. Although all these works investigated into the directional sensing models, none of which considered the visual occlusion problem among crowded targets.

For a target, to stand within the FOV of a visual sensor may not mean being captured by the camera because there may be other targets standing between the target and the camera and visually occluding them. [Lin et al. 2011] developed analytical expressions to derive expected coverage by a randomly deployed single camera in a sensing field that is occlusion free or with occlusion. Then, they extended this method to the expected joint coverage after deploying additional cameras into field iteratively. [Qian and Qi 2008] derived several parameters such as minimum, maximum and expectation values for visual coverage estimation in the presence of visual occlusions for VSNs. However, neither approach derived a closed-form solution for visual K-coverage probability estimation due to the target detection model used.

3. TARGET DETECTION MODEL

In traditional target detection algorithms, the intersections of the back-projected 2D visual cones of the targets are calculated to localize all the individual targets. If the cones from different sensor nodes intersect at the same point, it can be considered there is at least one target in that intersection. Existing coverage estimation algorithms are based on the information about the target “existence” at the intersections. However, this information cannot be certain since in crowded environments, many “empty” intersections that are not actually occupied by any target are created because of the visual occlusion (e.g., intersection D) or ghost positioning (e.g., intersection C), as shown in Fig. 1. In addition to this, existing coverage estimation approaches do not take the partial appearance of targets in the FOV of sensor nodes into account. However, in a crowded scene, it might not be realistic to have a free sight for all targets in the sensing field because of the visual occlusion among the crowd. Due to the uncertainty about real locations of the targets and partial appearance of targets, the derivation of a closed-form solution for the coverage estimation has been a very challenging problem. We refer to this traditional model as “uncertainty-based or occupancy-based target detection”.

In this section, we briefly describe an inverse approach to traditional target de-
Fig. 2: The certainty-based target detection model. (a) Visual hulls of targets in 3D, (b) Projection of 3D cones onto the ground plane, and (c) Certain areas about target non-existence (labeled with white).

detection problem proposed by the authors previously in [Karakaya and Qi 2011]. Instead of resolving the uncertainty about target existence, we identify and study the non-occupied areas in the visual cone. If an area within the FOV of a sensor node is not occupied or occluded by any object, it is certain about target non-existence and declared as a non-occupied area. Otherwise, it is uncertain about target existence in the corresponding region. The occupied areas are the ones where it is possible that there exist targets. The uncertainty is due to either occlusion or outside of the FOV of the camera. We refer to this model as the “certainty-based target detection”.

The certainty-based target detection model is illustrated in Fig. 2. Each target is modeled by a uniform size cylindrical object in 3D where texture and shape signatures of the target are contained within the cylinder space around the axis as shown in Fig. 2a. [Yang et al. 2003] showed that it is reasonable to model the objects of similar heights and widths using cylinders for people and vehicle detection algorithms. After background subtraction, each target can be extracted from the scene, which sweeps a cone in 3D space as shown in Fig. 2a. To find visual cone of the target, these 3D cones are projected onto a plane parallel to the ground as seen in Fig. 2b. The non-occupied (labeled with white) areas in the visual cone are thus determined as shown in Fig. 2c. The detailed comparison between certainty-based and the traditional occupancy-based target detection models and their impact on the closed-form solution for visual coverage estimation will be presented in Section 6.

In this paper, we assume that the infinitesimal visual sensor nodes with uniform FOV and sensing radius are randomly deployed within a very large two-dimensional sensing field, $R$. Since each region in the sensing field has equal importance based on the probability of target existence, all sensor nodes are uniformly and independently distributed into the sensing field. Based on this deployment strategy, the locations of visual sensor nodes can be modeled by a two-dimensional stationary Poisson point process with sensor density $\lambda_s$ [Wang et al. 2010]. It is also assumed that orientations of visual sensors are uniformly distributed over $[0^\circ, 360^\circ)$. Let $\rho$ and $\theta$
Table I: Notations

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>$R$</td>
<td>2D sensing field</td>
</tr>
<tr>
<td>$(x, y)$</td>
<td>Coordinates of a grid point in the sensing field $R$</td>
</tr>
<tr>
<td>$(u, v)$</td>
<td>Minimum distances from a grid point to two borders of the sensing field</td>
</tr>
<tr>
<td>$l$</td>
<td>Distance between the grid point and the sensor node</td>
</tr>
<tr>
<td>$A$</td>
<td>Circular detectability area</td>
</tr>
<tr>
<td>$A_o$</td>
<td>Area of the occlusion zone</td>
</tr>
<tr>
<td>$\lambda_t$</td>
<td>Target density</td>
</tr>
<tr>
<td>$\lambda_s$</td>
<td>Sensor density</td>
</tr>
<tr>
<td>$\tilde{\lambda}_s$</td>
<td>Minimum sensor density</td>
</tr>
<tr>
<td>$r$</td>
<td>Target radius</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Sensing range of a camera</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Angle of view of a camera</td>
</tr>
<tr>
<td>$p$</td>
<td>Probability of the sensor node facing towards the center of detectability area</td>
</tr>
<tr>
<td>$q$</td>
<td>Probability that there is no occlusion between a grid point and a node</td>
</tr>
<tr>
<td>$Q = p \times q$</td>
<td>Probability of covering a specific grid point of the sensing field</td>
</tr>
<tr>
<td>$P(k)$</td>
<td>Probability that exactly $k$ sensor nodes cover a specific grid point of the sensing field $R$</td>
</tr>
<tr>
<td>$P(k; Q)$</td>
<td>Probability that exactly $k$ sensor nodes cover a specific grid point of the sensing field $R$ with respect to $Q$</td>
</tr>
<tr>
<td>$P(j; \lambda_s \times A)$</td>
<td>Probability that a detectability area $A$ contains exactly $j$ sensor nodes from a Poisson point process with sensor density $\lambda_s$</td>
</tr>
<tr>
<td>$f(l)$</td>
<td>Probability density function (pdf) of distance $l$</td>
</tr>
<tr>
<td>$f(Q)$</td>
<td>Probability density function (pdf) of function $Q$</td>
</tr>
<tr>
<td>$F_P(k; \lambda)$</td>
<td>Cumulative distribution function (cdf) of Poisson distribution</td>
</tr>
<tr>
<td>$K$</td>
<td>K-coverage requirement, i.e., at least $K$ nodes covers a specific grid point</td>
</tr>
<tr>
<td>$\Gamma(k, \lambda)$</td>
<td>Upper incomplete gamma function with parameter $k$ and $\lambda$</td>
</tr>
<tr>
<td>$T(3, k, \lambda)$</td>
<td>A special case of Meijer G-function with parameter $k$ and $\lambda$</td>
</tr>
<tr>
<td>$E_1(\lambda)$</td>
<td>Exponential integral with parameter $\lambda$</td>
</tr>
</tbody>
</table>

denote, respectively, the sensing radius and angle of view of a sensor node.

Let us model a target as a uniform disc on the 2D plane, $R$, with radius, $r$, when the cylindrical object is projected onto a plane parallel to the ground. In addition, there is no overlap between the targets and sensors in $R$. We further assume that the centers of all existing targets in the scene are uniformly distributed which means that the probability of any point in $R$ to be occupied by a target is the same across the sensing field. Based on this random and uniform target distribution model, the probability that a number of target centers are located within a region, $A$, can be estimated by a two-dimensional stationary Poisson point process with a parameter $\lambda_t \times A$, where $\lambda_t$ and $A$ denote, respectively, the target density and the area of region $A$. For clarification purpose, we summarize the notations used in the derivation of the closed-form solution in Table I.

4. VISUAL COVERAGE WITHOUT VISUAL OCCLUSIONS

If the radius of targets is infinitely small, i.e., $r \to 0$, we can ignore the visual occlusion. That is, in the “certainty-based target detection”, all areas within the FOV of the sensor node would be marked as “white” (see Fig. 2c), for sensor coverage. In other words, a sensor node covers a specific grid point $(x, y) \in R$, of the sensing field and determines target non-existence, if the node is located in a circular area $A$ with radius $\rho$ centered at the corresponding grid point and is
oriented towards the center of the circle. In the rest of the paper, the circular area $A$ is referred to as the “detectability area”. Therefore, the visual coverage probability, defined as the probability that exactly $k$ sensor nodes cover a specific grid point of the sensing field and determine the target non-existence is

$$P(k) = \sum_{j=k}^{\infty} \mathcal{P}(j; \lambda_s \times A) \binom{j}{k} p^k (1 - p)^{j-k}$$

where $\mathcal{P}(j; \lambda_s \times A)$ denotes the probability that a detectability area $A$ contains exactly $j$ sensor nodes from a Poisson point process with sensor density $\lambda_s$, i.e., $\mathcal{P}(j; \lambda_s \times A) = e^{-\lambda_s \times A} / j!$ where $A = \pi r^2$. And, $p$ denotes the probability of the sensor node facing towards the center of detectability area, $A$, i.e., $p = \theta/(2\pi)$ and $\binom{j}{k}$ denotes the number of combinations of $k$-node subset from a $j$-node set.

Eq. 1 can be further derived as,

$$P(k) = \sum_{j=k}^{\infty} e^{-\lambda_s \times A} \binom{j}{k} \frac{j!}{k!(j-k)!} p^k (1 - p)^{j-k} = \frac{1}{k!} e^{-\lambda_s \times A} (\lambda_s \times A \times p)^k \sum_{j=k}^{\infty} (\lambda_s \times A(1-p))^{j-k} (j-k)!$$

$$= \frac{1}{k!} e^{-\lambda_s \times A} (\lambda_s \times A \times p)^k (\lambda_s \times A(1-p))^j$$

$$= \frac{1}{k!} e^{-\lambda_s \times A \times p} (\lambda_s \times A \times p)^k$$

$$= \mathcal{P}(k; \lambda_s \times A \times p)$$

From the derivation result in Eq. 2, we observe that the visual coverage probability without visual occlusions follows the Poisson point process with density $\lambda_s \times \theta/(2\pi)$ in the detectability area $A$.

5. VISUAL COVERAGE WITH VISUAL OCCLUSIONS

In an environment with crowded targets, it is no longer appropriate to assume an infinitely small target and target radius $r$ becomes a finite value, i.e., $r > 0$. Hence, visual occlusions should be taken into account. To cover a specific grid point of the sensing field and determine the target non-existence at that point, not only the corresponding grid point must be inside the FOV of the sensor node, the centers of all targets should also be outside of the occlusion zone between the corresponding grid point and the node, which is illustrated as the bold-boundary region in Fig. 3.

The shaded region in Fig. 3 is the area of the occlusion zone, denoted as $A_o$. The value of $A_o$ depends on the target radius $r$ and the distance $l$ between the corresponding grid point and visual sensor node and can be expressed as $A_o = \pi r^2 + 2rl$. Let $q$ denote the probability that there is no visual occlusion between the grid point and the sensor node. Since the probability that a number of target centers are located within a region, $A_o$, follows Poisson distribution, the probability of having no targets in the occlusion zone, $q$, equals to $e^{-\lambda_i (\pi r^2 + 2rl)}$ which is a random value with respect to the randomness of the distance $l$ between the grid point and the visual sensor node. Let $Q$ denote the probability of covering a specific
grid point of the sensing field and determining the target non-existence. Since the visual coverage depends on two independent factors, i.e., the grid point is within the FOV of the sensor and that there is no occlusion between the sensor and the grid point, \( Q \) can be expressed as

\[
Q(l) = p \times q = \frac{\theta}{2\pi} \times e^{-\lambda t(\pi r^2 + 2rl)} \tag{3}
\]

Thus, the visual coverage probability that exactly \( k \) nodes cover a specific grid point of the sensing field and determine the target non-existence, \( P(k) \), is

\[
P(k) = \int P(k, Q)f(Q)dQ \tag{4}
\]

where \( P(k, Q) \) is the probability that exactly \( k \) sensor nodes cover a specific grid point of the sensing field and determine the target non-existence at that point with respect to \( Q \), and \( f(Q) \) is the probability density function (pdf) of \( Q \) with respect to distance \( l_i \) between the corresponding grid point and each sensor node \( s_i \) in the circular detectability area \( A \) with radius \( \rho \) centered at the grid point, \( i = 1 \ldots N_s \), and \( N_s \) is the number of visual sensor nodes within area \( A \).

Since sensor nodes are uniformly distributed at random in the sensing field, the probability of sensor nodes appears at the same distance to the center of the circular detectability area \( A \) is proportional to the area of the region. Therefore, \( f(l) \), the pdf of distance \( l_i \) between the corresponding grid point and each sensor node \( s_i \), follows linear distribution from 0 to \( \rho \)

\[
f(l) = \begin{cases} 
\frac{2l}{\rho^2} & \text{for } 0 \leq l \leq \rho \\
0 & \text{otherwise}
\end{cases} \tag{5}
\]

To calculate the pdf of function \( Q(l) \), \( f(Q) \), we utilize the change of variable property. Since \( Q(l) \) is a monotonically decreasing function, \( f(Q) \) is

\[
f(Q) = \begin{cases} 
\frac{2}{\rho^2} \times \ln \left( \frac{\frac{2l}{\rho^2} e^{-\lambda t r^2}}{Q} \right) \times \frac{1}{2\lambda_t r Q} & \text{for } Q(l = \rho) \leq Q \leq Q(l = 0) \\
0 & \text{otherwise}
\end{cases} \tag{6}
\]
P(k,Q) can be derived as

\[ P(k,Q) = \sum_{j=k}^{\infty} \sum_{i=k}^{j} \mathcal{P}(j; \lambda_s \times A) C_i^j p^j (1-p)^{j-i} C_i^k q^k (1-q)^{i-k} \]

\[ = \sum_{j=k}^{\infty} \sum_{i=k}^{j} \mathcal{P}(j; \lambda_s \times A) p^k q^j (1-p)^{j-k} C_i^j C_i^k \left( \frac{p(1-q)}{1-p} \right)^{i-k} \]

\[ = \sum_{j=k}^{\infty} \sum_{i=k}^{j} \mathcal{P}(j; \lambda_s \times A) p^k q^j (1-p)^{j-k} \sum_{s=0}^{j-k} C_i^s C_i^{j-i-s} \left( \frac{p(1-q)}{1-p} \right)^s \]

\[ = \sum_{j=k}^{\infty} \sum_{i=k}^{j} \mathcal{P}(j; \lambda_s \times A) p^k q^j (1-p)^{j-k} C_i^j \left( \frac{p(1-q)}{1-p} + 1 \right)^{j-k} \]

\[ = \sum_{j=k}^{\infty} \sum_{i=k}^{j} \mathcal{P}(j; \lambda_s \times A) p^k q^j (1-p)^{j-k} \frac{j!}{k!(j-k)!} \left( \frac{1-pq}{1-p} \right)^{j-k} \]

\[ = \frac{1}{k!} e^{-\lambda_s \times A} \left( \lambda_s \times A \right)^k \sum_{j=k}^{\infty} \left( \frac{\lambda_s \times A}{1-pq} \right)^{j-k} \frac{1}{(j-k)!} = \mathcal{P}(k; \lambda_s \times A \times Q), \] 

\[ (7) \]

where (a) follows the combination properties, \( C_i^j C_i^{j+s} = C_i^j C_i^{j-k} \), and (b) follows the binomial coefficient property, \((x+y)^n = \sum_{s=0}^{\infty} C_n^s x^s y^{n-s}\) where \(s = i-k\). Also, \(A = \pi \rho^2\), \(p = \theta/(2\pi)\), \(q = e^{-\lambda_s (\pi r^2 + 2\pi l)}\), and \(Q = p \times q\).

From the derivation result in Eq. 7, we observe that the visual coverage probability with visual occlusions also follows the Poisson point process with the sensor density \(\lambda_s \times \theta/(2\pi) \times e^{-\lambda_s (\pi r^2 + 2\pi l)}\) in area \(A\). If \(\lambda_s \rightarrow 0\) or \(r \rightarrow 0\), then \(Q \rightarrow p = \theta/(2\pi)\) which means visual occlusions among the targets can be ignored. Therefore, Eq. 7 converges to Eq. 2. The derivation of the visual coverage probability that exactly \(k\) sensor nodes cover a specific grid point of the sensing field and determine the target non-existence at that point can then be derived as

\[ P(k) = \int P(k,Q) f(Q) dQ \]

\[ = \int \mathcal{P}(k; \lambda_s \times A \times Q) f(Q) dQ \]

\[ = \int_{Q(l=0)}^{Q(l=p)} \frac{\lambda_s AQ}{k!} \left( \frac{\lambda_s AQ}{Q} \right)^k \times 2 \rho^2 \times \frac{\ln \left( \frac{\pi e^{-\lambda_s \pi r^2}}{Q} \right)}{4\lambda_s^2 \pi^2 Q} dQ \] 

\[ (8) \]
Let \( u = \lambda_s \pi \rho^2 Q \), \( \lambda_c = \lambda_s \pi \rho^2 \frac{\theta}{\pi^2} e^{-\lambda_s \pi \rho^2} \) and \( \lambda_b = \lambda_c \times e^{-2 \lambda_s \pi \rho} \)

\[
P(k) = \frac{1}{2k! \rho^2 r^2 \lambda_r^2} \int_{u_Q(\theta=0)}^{u_Q(\theta=\rho)} u^k e^{-u} \times \frac{\ln \left( \frac{\pi e^{-\lambda_s \pi^2 \rho^2}}{u} \right)}{\lambda_s \pi \rho^2} \times du \]
\[
= \frac{1}{2k! \rho^2 r^2 \lambda_r^2} \int_{\lambda_b}^{\lambda_b} u^{k-1} e^{-u} \ln \left( \frac{\lambda_c}{u} \right) du
\]
\[
= \frac{1}{2k! \rho^2 r^2 \lambda_r^2} \left[ \int_{\lambda_b}^{\lambda_b} u^{k-1} e^{-u} \ln \lambda_c du - \int_{\lambda_c}^{\lambda_b} u^{k-1} e^{-u} \ln u du \right]
\]  
\[
= \ln \lambda_c \left[ \Gamma(k, \lambda_b) - \Gamma(k, \lambda_c) \right] 
\]  
(9)

where

\[ P_1(k) = \ln \lambda_c \int_{\lambda_b}^{\lambda_b} u^{k-1} e^{-u} du 
\]
\[ = \ln \lambda_c \left[ \int_{\lambda_b}^{\infty} u^{k-1} e^{-u} du - \int_{\lambda_c}^{\infty} u^{k-1} e^{-u} du \right] 
\]
\[ = \ln \lambda_c \left[ \Gamma(k, \lambda_b) - \Gamma(k, \lambda_c) \right] 
\]  
(10)

and

\[ P_2(k) = \int_{\lambda_b}^{\lambda_b} u^{k-1} e^{-u} \ln u du 
\]
\[ = \int_{\lambda_b}^{\infty} u^{k-1} e^{-u} \ln u du - \int_{\lambda_c}^{\infty} u^{k-1} e^{-u} \ln u du 
\]
\[ = \frac{\partial}{\partial k} \Gamma(k, \lambda_b) - \frac{\partial}{\partial k} \Gamma(k, \lambda_c) 
\]  
(\(a\)) \[ \ln \lambda_b \Gamma(k, \lambda_b) + \lambda_b \Gamma(3, k, \lambda_b) \]
\[ = \left[ \ln \lambda_c \Gamma(k, \lambda_c) + \lambda_c \Gamma(3, k, \lambda_c) \right] - \left[ \ln \lambda_b \Gamma(k, \lambda_b) + \lambda_b \Gamma(3, k, \lambda_b) \right] 
\]

where \( \Gamma(k, \lambda) \) is the upper incomplete gamma function and \( \frac{\partial}{\partial k} \Gamma(k, \lambda) \) is its first derivative with respect to \( k \) [Abramowitz 1970]. In this derivation, (a) follows \( \frac{\partial}{\partial k} \Gamma(k, \lambda) = \ln \lambda \Gamma(k, \lambda) + \lambda T(3, k, \lambda) \) \( k \)'s a special case of Meijer G-function [Geddes et al. 1990]. By using the simple recurrence formula of function \( T(3, k, \lambda) \), we can find its generalized recurrence formula as \( \lambda T(3, k, \lambda) = (k - 1)! E_1(\lambda) + \sum_{i=1}^{k-1} \frac{k-1}{i} \Gamma(i, \lambda) \) where \( E_1(\lambda) \) is the exponential integral. These special cases of function \( T(3, k, \lambda) \) provide an extension of \( P_2(k) \) as

\[
P_2(k) = \left[ \ln \lambda_b \Gamma(k, \lambda_b) + (k - 1)! E_1(\lambda_b) + (k - 1)! \sum_{i=1}^{k-1} \frac{\Gamma(i, \lambda_b)}{i!} \right] - \left[ \ln \lambda_c \Gamma(k, \lambda_c) + (k - 1)! E_1(\lambda_c) + (k - 1)! \sum_{i=1}^{k-1} \frac{\Gamma(i, \lambda_c)}{i!} \right] 
\]

(11)

By substituting Eqs. 10 and 11 into Eq. 9, a closed-form solution of the visual...
coverage probability, $P(k)$ can be further derived as,

$$P(k) = \frac{1}{2k!p^2\rho^2\lambda^2} \left[ P_1(k) - P_2(k) \right]$$

$$= \frac{1}{2k!p^2\rho^2\lambda^2} \left[ \ln \left( \frac{\lambda_c}{\lambda_o} \right) \Gamma(k, \lambda_b) - (k-1)! \left( E_1(\lambda_b) - E_1(\lambda_c) \right) - (k-1)! \sum_{i=1}^{k-1} \frac{\Gamma(i, \lambda_b) - \Gamma(i, \lambda_c)}{i!} \right]$$

$$= \frac{1}{2k!p^2\rho^2\lambda^2} \left[ \ln \left( \frac{\lambda_c}{\lambda_o} \right) F_p(k-1, \lambda_b) - \left( E_1(\lambda_b) - E_1(\lambda_c) \right) - \sum_{i=1}^{k-1} \frac{F_p(i-1, \lambda_b) - F_p(i-1, \lambda_c)}{i!} \right] \quad (12)$$

where (a) follows $\Gamma(a, x) = \int_x^\infty e^{-t} t^{a-1} dt$. When $a$ is an integer, $\Gamma(n, x) = (n-1)!e^{-x} \sum_{j=0}^{n-1} \frac{x^j}{j!}$ [Press et al. 2007]. In this derivation, $F_p(k; \lambda)$ is the cumulative probability distribution (cdf) of Poisson distribution with parameter $\lambda$.

In this paper, it is assumed that homogeneous visual sensor nodes with the same sensing radius, $\rho$ and angle of view, $\theta$ are deployed into the sensing field to detect the homogeneous targets with uniform target radius, $r$. In order to make the scenario more realistic where heterogeneous visual sensors and targets are likely to be deployed, we can relax these assumptions by considering the heterogeneous sensor deployment and heterogeneous target existence in the sensing field.

In the heterogeneous visual sensor deployment, we deploy different types of visual sensor nodes into the sensing field with different sensor density, $\lambda_s$, sensing radius, $\rho$, and angle of view, $\theta$. For instance, if two types of sensor nodes are deployed into the sensing field with sensor density, $\lambda_{s_1}$ and $\lambda_{s_2}$, sensing radius, $\rho_1$ and $\rho_2$, and angle of view, $\theta_1$ and $\theta_2$, a target can be covered by $k$ many sensor nodes with any combinations of Type I and Type II sensor nodes. Therefore, in the derivation of the closed-form solution for visual coverage estimation, we have to consider the different detection probability of each type of sensor node in their different size of detectability area, $A$ and derive the closed-form solution based on their sensor related parameters (i.e., $\lambda_s$, $\rho$, and $\theta$). The probability of Type I sensor nodes facing towards the center of their detectability area, $A_1$ is $p_1 = \theta_1/(2\pi)$ where $A_1 = \pi\rho_1^2$ and probability of Type II sensor nodes facing towards the center of their detectability area, $A_2$ is $p_2 = \theta_2/(2\pi)$ where $A_2 = \pi\rho_2^2$.

In addition, when heterogeneous targets exist in the sensing field with different target radius, $r$, we have to consider all types of targets in order to compute the probability that there is no occlusion between a grid point and a sensor node, $q$. For example, if two types of targets exist in the sensing field with target density, $\lambda_t$ and $\lambda_{t_2}$, and target radius, $r_1$ and $r_2$, to cover a grid point in the sensing field, the occlusion zones between the grid point and the sensor node, $A_{o_1}$ and $A_{o_2}$ have to be free of Type I and Type II targets, respectively. Therefore, the probability
of having no Type I target in the occlusion zone, $A_{o1}$ equals to $e^{-\lambda_1 A_{o1}}$ where $A_{o1} = \pi r_1^2 + 2r_1 l$ and the probability of having no Type II target in the occlusion zone, $A_{o2}$ equals to $e^{-\lambda_2 A_{o2}}$ where $A_{o2} = \pi r_2^2 + 2r_2 l$. The complete analysis of the heterogeneous visual sensor node deployment and heterogeneous target existence in VSNs will be part of our future work.

6. COMPARISON BETWEEN OCCUPANCY-BASED AND CERTAINTY-BASED VISUAL COVERAGE ESTIMATION

In this section, we explicitly discuss the advantages of adapting the certainty-based target detection approach that integrates target non-existence information versus the occupancy-based target detection approach that integrates target existence information in deriving the closed-form solution for visual coverage estimation in VSNs with visual occlusions.

In traditional occupancy-based target detection algorithms, since the intersections of the back-projected 2D visual cones of the targets are calculated to localize all the individual targets, occupancy maps hold the information about total number of sensor nodes detecting target existence at each intersection. Therefore, existing coverage estimation algorithms are based on the occupancy information about the target “existence” at the intersections and define the target detection only if the front arc of the disc bounded by two tangent viewing rays is completely visible to the sensor node, as shown in Fig. 4a. More specifically, in the occupancy-based model, to declare target existence at a specific intersection, not only the arc of the disc bounded by tangent rays must be inside the FOV of the sensor node, the centers of all other targets should also be outside of the occlusion zone between the corresponding target and the sensor node, which is illustrated as the bold-boundary region in Fig. 4a.

The shaded regions in Fig. 5 are the occupancy-based occlusion zones, denoted as $A_o$ which are random values with respect to the randomness of the distance $l$ between the target and the sensor node. Since the visual coverage depends on two independent factors, i.e., the target is within the FOV of the sensor and that there is no occlusion between the sensor and the target, the probability of covering a
specific target, $Q$, can be
\[ Q(l) = p \times q = \frac{\theta - 2 \arcsin(r/l)}{2\pi} \times e^{-\lambda_t \times A_o}. \]

where $p$ denotes the probability of the sensor node facing towards the target, i.e., $p = \left(\theta - 2 \arcsin(r/l)\right)/(2\pi)$ and $q$ denotes the probability of having no targets in the occlusion zone i.e., $q = e^{-\lambda_t \times A_o}$.

However, Eq. 13 is valid if and only if free sight is available for all targets in the sensing field as illustrated as the bold-boundary regions in Fig. 5a. In a crowded scene, it is not appropriate to assume the existence of a free sight for all targets in the sensing field because of the visual occlusions among targets. Since some targets partially appear in the FOV of sensor nodes, the area of the occlusion zone, $A_o$, becomes a random variable with respect to not only the distance $l$ but also the location of other occluding targets as illustrated as the bold-boundary regions in Fig. 5b. Therefore, the probability density function (pdf) of $Q$, $f(Q)$ could not be calculated explicitly. As a result, due to the partial appearance of targets, the derivation of a closed-form solution for the occupancy-based coverage estimation has been a very challenging problem in VSNs.

To solve this problem, we adopt certainty-based target detection model in the derivation of visual coverage estimation in VSNs. Instead of resolving the uncertainty about target existence, we identify and study the non-occupied areas in the visual cone to detect targets. When the non-existence information coming from different sensor nodes is fused in a certainty map to remove the uncertainty, the only uncertain regions left would be the location of targets. In other words, certainty maps hold the information about total number of sensor nodes detecting target non-existence at any specific region. Therefore, in the certainty-based approach, to cover a specific grid point of the sensing field and determine the target non-existence at that point, not only the corresponding grid point must be inside the FOV of the sensor node, the centers of all targets should also be outside of the occlusion zone between the corresponding grid point and the node, which is illustrated as the bold-boundary region in Fig. 4b.
Fig. 6: (a) Partitioned boundary sub-regions of a rectangular sensing field and (b) detectability areas in (top) corner sub-region $A_C$, (bottom) side sub-region $A_S$.

Unlike the occupancy-based occlusion zone model, the area of the certainty-based occlusion zone, $A_o$, depends on only the distance $l$ between the corresponding grid point and sensor node and can be expressed as $A_o = \pi \rho^2 + 2\pi \rho l$. As described in Section 5, the calculation the pdf of function $Q(l)$, $f(Q)$, is utilized by the change of variable property on function $f(l)$, the pdf of distance $l$. Therefore, the certainty-based model enables the computation the pdf of function $Q(l)$, $f(Q)$. As a result, the derivation of the closed-form solution for visual coverage estimation in VSNs has been possible by adopting the certainty-based target detection model.

7. BOUNDARY EFFECT ON THE VISUAL COVERAGE ESTIMATION

In this section, we investigate the boundary effect on the visual coverage estimation. For a sensor node close to the boundary of the sensing field, part of the area within its FOV will fall outside of the sensing field $R$. Therefore, the visual sensor coverage probability at the boundary of the sensing field is less than that in central areas of the sensing field. This is commonly referred to as the boundary effect in sensor networks.

As shown in the derivation of Eq. 12, the visual coverage probability $P(k)$ denotes the probability that a specific grid point within the sensing field $R$ is covered by exactly $k$ many visual sensor nodes out of $j$ many nodes distributed in a circular detectability area $A$ with radius $\rho$ centered at the grid point. If the boundary effect is ignored, $A = \pi \rho^2$ holds for all grid points within the sensing field, $R$, so $P(k)$ is similar at all points in $R$ as well. However, due to the boundary effect, the grid points close to the boundary have a partial circular detectability area $A(x, y)$, shown as gray regions in Fig. 6 and $A(x, y) \leq \pi \rho^2$. Therefore, visual coverage probability $P(k)$ depends on the location in the sensing field $R$.

[Yen et al. 2006] discussed region partitioning to estimate the boundary effect on the expected coverage in wireless sensor networks according to the locations of omnidirectional scalar sensors. Following the similar partitioning idea but with different partitioning approaches for visual sensor networks, we divide the sensing field $R$ into three types of sub-regions according to the location of grid point $(x, y)$. ACM Journal Name, Vol. V, No. N, 05 2011.
Let \(A^C, A^S, A^M\) represent the sub-regions where a grid point is located in the corner sub-regions, side sub-regions and middle sub-regions of the sensing field \(R\), respectively.

As shown in Fig. 6a, the detectability area of each grid point in the middle sub-region \(A^M\) has circular shape because distances of a grid point to the two closest borders of the sensing field \(R\) is more than the sensing range of the sensor, \(\rho\). Therefore, \(A^M(x, y) = \pi \rho^2, \forall (x, y) \in A^M\). In the following subsections, we estimate the detectability area of the corner sub-regions \(A^C\) and side sub-regions \(A^S\) within the sensing field \(R\).

7.1 Computing Detectability Area at the Corner Sub-region \(A^C\)

Fig. 6b (top) illustrates the detectability area of a grid point in a corner sub-region \(A^C\) where the distance of a grid point to the closest corner is less than the sensing range of a visual sensor, \(\rho\). Let \(u, v\) denote the minimum distances from a grid point in a corner sub-region \(A^C\) to two borders of the \(M \times N\) rectangle sensing field \(R^{M \times N}\), respectively, i.e., \(u = \min(x, M - x), v = \min(y, N - y)\) and \(u^2 + v^2 \leq \rho^2\).

By geometry, the detectability area of a grid point in a corner sub-region \(A^C(x, y)\) is expressed as,

\[
A^C(x, y) = u \times v + \frac{u \sqrt{\rho^2 - u^2}}{2} + \frac{v \sqrt{\rho^2 - v^2}}{2} + \left(\frac{\pi}{2} + \arcsin\left(\frac{u}{\rho}\right) + \arcsin\left(\frac{v}{\rho}\right)\right)\pi \rho^2
\]

(14)

Thus, the detectability area of a grid point in a corner sub-region \(A^C\) decreases as the point is located closer to the corner of the sensing field \(R\). Based on the decrease in the detectability area, the visual coverage probability \(P(k)\) decreases accordingly.

7.2 Computing Detectability Area at the Side Sub-region \(A^S\)

Fig. 6b (bottom) illustrates two types of detectability areas of grid points in a side sub-region \(A^S\) where at least one of the distances between a grid point and the two closest borders of the sensing field \(R\) is less than the sensing range of a visual sensor, \(\rho\). Let \(u, v\) denote the minimum distances from a grid point in a side sub-region \(A^S\) to borders of the \(M \times N\) rectangle sensing field \(R^{M \times N}\), respectively, i.e., \(u = \min(x, M - x), v = \min(y, N - y)\), \(u^2 + v^2 > \rho^2\) and \(u \leq \rho\) or \(v \leq \rho\). By geometry, three types of detectability area of a grid point in a side sub-region \(A^S(x, y)\) can be expressed as,

\[
A^S(x, y) = \begin{cases} 
  u \sqrt{\rho^2 - u^2} + v \sqrt{\rho^2 - v^2} + \frac{2\pi - 2 \arccos\left(\frac{u}{\rho}\right) - 2 \arccos\left(\frac{v}{\rho}\right)}{2\pi} \pi \rho^2, & \text{if } u \leq \rho \text{ and } v \leq \rho \\
  u \sqrt{\rho^2 - u^2} + \frac{2\pi - 2 \arccos\left(\frac{u}{\rho}\right)}{2\pi} \pi \rho^2, & \text{if } u \leq \rho \text{ and } v > \rho \\
  v \sqrt{\rho^2 - v^2} + \frac{2\pi - 2 \arccos\left(\frac{v}{\rho}\right)}{2\pi} \pi \rho^2, & \text{if } u > \rho \text{ and } v \leq \rho
\end{cases}
\]

(15)

Thus, the detectability area of a grid point in a side sub-region \(A^S\) decreases as the point gets closer to the borders of the sensing field \(R\). Based on the decrease in the detectability area, the visual coverage probability \(P(k)\) decreases in a side sub-region \(A^S\) accordingly.

8. MINIMUM SENSOR DENSITY ESTIMATION

In many visual sensor deployment applications, one of the major tasks is to find accurate estimation of the minimum sensor density to deploy into the sensing field which is sufficient to ensure the visual coverage probability that each point is covered by at least K sensor nodes is higher than a certain percentage. In other words, the probability that each point is covered by less than K sensor nodes is smaller than a tolerance value $\varepsilon$. The optimization problem of minimum node density to ensure visual K-coverage can be expressed as,

$$\hat{\lambda}_s = \arg \min_{\lambda_s \geq 0} \left| \sum_{k=0}^{K-1} P(k) - \varepsilon \right|$$

where $P(k)$ is parameterized by $\lambda_s$ and other fixed parameters $\lambda_t$, $r$, $\rho$, and $\theta$ as shown in Eq. 12. Therefore, the solution for optimization problem is that minimum node density is the smallest positive root $\hat{\lambda}_s$ of the following equation

$$K-1 \sum_{k=0}^{K-1} \frac{1}{2k^2 r^2 \lambda_t^2} \left[ \ln \left( \frac{\lambda_c}{\lambda_b} \right) F_p(k - 1, \lambda_b) - \left( E_1(\lambda_b) - E_1(\lambda_c) \right) - \sum_{i=1}^{k-1} F_p(i - 1, \lambda_b) - F_p(i - 1, \lambda_c) \right] = \varepsilon$$

where $\lambda_c$ and $\lambda_b$ are the Poisson distribution parameters, i.e., $\lambda_c = \lambda_s \pi \rho^2 \theta / 2 \pi e^{-\lambda_t \pi r^2}$ and $\lambda_b = \lambda_s \pi \rho^2 \pi / 2 \pi e^{-\lambda_t (\pi r^2 + 2\lambda_t \rho r)}$. There is no explicit solution for Eq. 17, so minimum sensor density, $\hat{\lambda}_s$ can be found by using the exhaustive search method.

9. EXPERIMENTS AND RESULTS

In this section, we first present the comparison between the simulation results and theoretical values to validate the theoretical derivation of visual coverage probability. Then, the results of minimum sensor density $\hat{\lambda}_s$ are presented to show the effect of visual occlusions among crowded targets on the visual coverage probability that ensures the K-coverage in the sensing field.

In our simulations, circular targets with uniform size are deployed on a 2D sensing field, infinitely small-size sensor nodes with uniform FOV and focal length are located and directed horizontally facing the sensing field. The locations of each sensor node and target are randomly generated assuming there is no overlap between the targets and sensor nodes. The orientation of each node is a floating point number randomly generated in $[0^\circ, 360^\circ]$. In all the simulations, we assume each sensor node is able to find its orientation and location by using a digital compass and positioning system, such as GPS.

Following is the setup of some typical parameters: The 2D sensing field is 40m x 40m large. The radius of each target, $r$ is 0.5m. Each sensor node has a uniform FOV with $\rho = 10m$ of sensing range and $\theta = 45^\circ$ of angle of view. Each node is in the communication range of other nodes and is able to communicate with each other. Fig. 7 illustrates a sample random deployment of 20 targets, represented as discs, and 100 cameras, represented as points.
Fig. 7: Simulation setup with 20 targets and 100 sensor nodes.

We conduct two sets of experiments to validate the theoretical derivation of visual coverage probability, where one set does not consider the boundary effect and the other one does. We also show the effect of parameter selection on the minimum sensor density, $\lambda_s$. In each set of experiments, different amount of coverage requirements $K$ ($K = 1, 2, 3$) are selected for ten times and the results are averaged.

9.1 Comparison between Theoretical Values and Experimental Results Without Boundary Effects

In this set of experiments, boundary effect is not considered. The effects of two groups of parameters are studied, including sensor node related parameters and target related parameters.

9.1.1 Effect of Sensor Node Related Parameters. In this experiment, we study the effect of the sensor node related parameters, sensor node density $\lambda_s$, sensing range $\rho$, and angle of view $\theta$ on the visual coverage probability for different visual K-coverage requirements.

In the first simulation, 20 targets and different numbers of sensor nodes are randomly deployed into the sensing field. Fig. 8a shows the visual coverage probability for different K values corresponding to different numbers of sensor nodes, $N_s$. We observe that visual coverage probability decreases as K increases because of the more demanding coverage requirement. In addition, visual coverage probability increases as $N_s$ increases due to more dense visual sensor nodes deployed.

Secondly, fixed number of sensor nodes and fixed number of targets are deployed into the sensing field where $N_s = 100$ and $N_t = 20$. However, in each deployment, we vary the value of the uniform sensing range $\rho$ of every visual sensor node. Fig. 8b shows the visual coverage probability for different K values corresponding to different sensing range $\rho$. We observe that visual coverage probability increases as $\rho$ increases or $K$ decreases because of larger visual coverage of each sensor node with larger FOV and less demanding coverage requirements, respectively.

In the third simulation, we select different values for the angle of view $\theta$ of each sensor node to show its effect on the visual coverage probability where fixed number of targets and fixed number of sensor nodes with fixed sensing range $\rho$ are deployed into the sensing field, i.e. $N_t = 20$, $N_s = 100$ and $\rho = 10m$. Fig. 8c shows the visual coverage probability for different K values corresponding to different angles of view $\theta$. We observe that visual coverage probability increases as $\theta$ increases or K decreases because of, again, larger visual coverage of each sensor node with larger FOV.

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Experimental results
Theoretical results

(a)

(b)

(c)

Fig. 8: Comparison of theoretical values and simulation results corresponding to sensor node related parameters, (a) different numbers of sensor nodes $N_s$, (b) different sensing range $\rho$ and (c) different angle of views $\theta$.

FOV and less demanding coverage requirements, respectively.

As shown in Fig. 8(a-c), the simulated experimental results are consistent with the theoretical values. However, because of the boundary effect, we observe that the visual K-coverage probability resulted from simulated experiments are slightly less than theoretical values. Moreover, the difference between theoretical values and experimental results increases as either $N_s$ or $\rho$ or $\theta$ increases because the boundary effect is more severe.

9.1.2 Effect of Target Related Parameters. In this experiment, we study the effect of the target related parameters, the number of deployed target $N_t$ and target radius $r$ on the visual coverage probability for different visual K-coverage.

First, different numbers of target, $N_t$, are deployed in the sensing field where the total number of cameras, $N_s$, is fixed at 100. Fig. 9a shows the visual coverage probability for different $K$ values corresponding to different numbers of deployed targets, $N_t$. We observe that visual coverage probability decreases as either $N_t$ or $K$ increases because of presence of more visual occlusions among more dense targets and more demanding coverage requirements, respectively.
In the second simulation, fixed number of sensor nodes and fixed number of targets are deployed where \( N_s = 100 \) and \( N_t = 20 \). However, in each deployment uniform radius of each target is chosen with different values to show the effect of the target radius \( r \). Fig. 9b shows the visual coverage probability for different \( K \) values corresponding to different target radius, \( r \). We observe that visual coverage probability decreases as either \( r \) or \( K \) increases because of the presence of more visual occlusions among bigger targets and more demanding coverage requirements, respectively.

Results in Fig. 9(a-b) further validate the theoretical derivation of visual sensor coverage by showing the consistent theoretical values with simulated experimental results. However, again due to the boundary effect, the visual coverage probability from simulated experiments shows slight difference from theoretical values.

### 9.2 Boundary Effect on the Coverage Estimation Probability

In this experiment, we study the boundary effect on the visual coverage estimation probability corresponding to different number of sensor nodes \( N_s \) for different visual \( K \)-coverage requirements. Fig. 10a shows the visual \( K \)-coverage probability in 2D sensing region \( R \) corresponding to \( N_t = 20, N_s = 100, \rho = 10, \theta = 45^\circ \) and \( K = 2 \). We observe that the visual \( K \)-coverage probability decreases in the boundary region as close to the edge of the sensing region because of the boundary effect.

We randomly deploy 20 targets and different numbers of sensor nodes into the sensing field. Fig. 10b shows the visual coverage probability of simulated experiment, theoretical results with and without boundary effect for different \( K \) values. We observe that visual coverage probability decreases as \( K \) increases because of the more demanding coverage requirement and visual coverage probability increases as \( N_s \) increases due to more dense sensor nodes.

Results in Fig. 10b validate the proposed theoretical derivation of visual sensor coverage by showing exactly the same theoretical values when boundary effect is taken into account with simulated experimental results.
9.3 Minimum Sensor Density

In this set of simulation results, we compute the minimum sensor density, $\hat{\lambda}_s$, that ensures visual K-coverage. We study the effect of different parameters, i.e. target density $\lambda_t$, target radius $r$, sensing range $\rho$ and angle of view $\theta$. In each experiment, we change the value of one of these parameters and fix other parameters by setting $\lambda_t = 0.1$, $r = 0.5m$, $\rho = 10m$, $\theta = 45^\circ$ and tolerance value $\epsilon = 0.05$.

First of all, we change the number of deployed target, $N_t$ from 10 to 300. Fig. 11a shows the minimum sensor density $\hat{\lambda}_s$ corresponding to different number of deployed target, $N_t$ under different K-coverage requirements. We observe that $\hat{\lambda}_s$ increases as $N_t$ increases because of the presence of more visual occlusions among more dense targets; $\hat{\lambda}_s$ also increases as K increases due to the more demanding coverage requirements. Moreover, we observe that to get double the K-coverage requires less than double increment in the sensor density because of the overlapping FOV of sensor nodes. However, to get more K-coverage in a crowded environment requires more proportional increment in the minimum sensor density $\hat{\lambda}_s$ than in a sparse target environment because of the more occlusion among crowded targets.

Secondly, to show the effect of the target radius $r$ on the minimum sensor density, we change its value from 0.1m to 5m. Fig. 11b shows the minimum sensor density $\hat{\lambda}_s$ corresponding to different target radius $r$. We observe that $\hat{\lambda}_s$ increases as either $\lambda_t$ or K increases because of the presence of more visual occlusions among targets of larger size and more demanding coverage requirements, respectively. Moreover, we observe that to update K-coverage requires less than K times increment in the sensor density. However, in an environment with large size targets, it requires more proportional increment in the minimum sensor density $\hat{\lambda}_s$ than in an environment with small-size targets because of the more visual occlusion among large targets.

In the following two simulations, we change the values of the sensor related parameters (i.e., sensing range $\rho$ from 3m to 20m and angle of view $\theta$ from $10^\circ$ to $120^\circ$) to study their effect. In Fig. 11c, the minimum sensor density $\hat{\lambda}_s$ is shown corresponding to different sensing range $\rho$. Fig. 11d shows the minimum sensor density $\hat{\lambda}_s$ corresponding to different angle of view $\theta$.
density $\hat{\lambda}_s$ corresponding to different angle of view $\theta$. We observe that $\hat{\lambda}_s$ decreases as either $\rho$ or $\theta$ increases due to the larger FOV of each sensor node and $\hat{\lambda}_s$ increases as $K$ increases because of more demanding coverage requirements.

10. CONCLUSION AND FUTURE WORK

In this paper, we presented a closed-form solution for the visual coverage estimation problem in the presence of visual occlusions among crowded targets in a VSN. By assuming the uniform random deployment of sensor nodes into a large-scale sensing field and taking the visual occlusions and boundary effects into account, we derived the visual coverage estimation from a different point of view by modeling the target detection algorithm based on the certainty map approach. Then, we further estimated the minimum sensor density that suffices to ensure a visual $K$-coverage in a crowded sensing field by using the visual coverage estimation model.

Our major contributions in this paper were two-fold. First, we adopted the certainty-based target detection model in coverage estimation in a randomly deployed VSN and derived a closed-form solution for visual coverage estimation. Therefore, the sensor related parameters (e.g., sensor density, sensing range, etc.)
can be decided before deployment in order to have proper visual coverage in the sensing field. Second, since the visual coverage probability in a crowded environment depends not only on the sensor density and deployment but also on the target density and distribution, our proposed closed-form solution considers both the directional sensing nature of cameras and the visual occlusions among targets and provides more accurate and more realistic coverage estimation in a crowded VSN.

By comparing the simulation results and the theoretical values, we validated the proposed closed-form solution of visual coverage estimation and showed the effectiveness of our model to be deployed in practical scenarios.

As part of future work, we would like to relax the assumptions on homogeneous sensor deployment and homogeneous target existence and extend the proposed closed-form solution for more general scenarios where heterogeneous visual sensors and targets are likely to be deployed into the sensing field.

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