Multiple Bits Distributed Moving Horizon State Estimation for Wireless Sensor Networks

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Outline

- Background
- Problem Statement
- Main Results
- Simulation Study
- Conclusion

Background

- Wireless sensor network (WSN) could be used to perform advanced signal processing tasks such as distributed detection and estimation.
- Environment monitoring and target tracking.
- Under bandwidth constraints → quantization, only quantized observations are communicated.
- Dynamic systems state estimation.

Deterministic parameter estimators

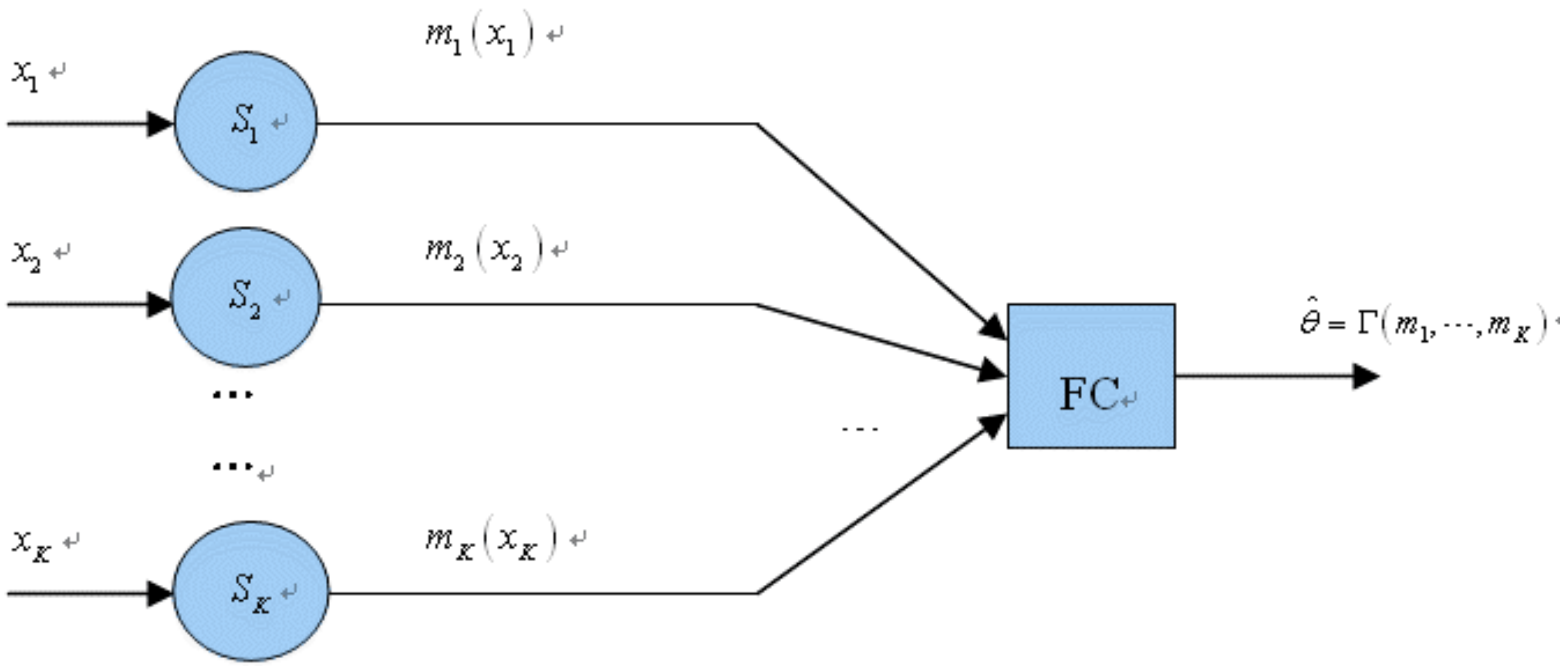
- Ribeiro and Giannakis etc. studied problems of maximum liklihood estimation under various noise assumption and signal and noise ratio (SNR) condition.
- Xiao and Luo etc. designed decentralized linear estimation schemes that do not require the knowledge of noise pdf, the estimator can be used in large scale WSN.

Distributed estimation framework

- Consider a generic distributed estimation problem using a WSN with an fusion center (FC)

$$x_k = \Phi_k(\theta) + w_k, \qquad k = 1, \cdots, K$$

where $\Phi_k : \mathbb{R}^p \rightarrow \mathbb{R}$ is generally a nonlinear function and the noise terms $w_k, k = 1, \cdots, K$ are zero-mean independent random variables.



Distributed estimation setup.

If there is no quantization

- Consider the simple signal model:

$$x_k = \theta + w_k \qquad k = 1, \cdots, K$$

- The best linear unbiased estimator (BLUE):

$$\hat{\theta}_{BLUE} = \sum_{k=1}^{K} x_k / \sigma_k^2 \Big/ \sum_{k=1}^{K} 1/\sigma_k^2$$

- Mean-square error (MSE):

$$E\left\{\left(\hat{\theta}_{BLUE} - \theta\right)^2\right\} = \left(\sum_{k=1}^{K} 1/\sigma_k^2\right)^{-1}$$

- Especially, when $\sigma_k^2 = \sigma^2$, $\hat{\theta}_{BLUE} = \sum_{k=1}^{K} x_k \Big/ K$, whose MSE is known to be σ^2/K.

Example Ⅰ: ε -estimator

- Consider a set of *K* distributed sensors with observations

$$x_k = \theta + w_k \qquad k = 1, \cdots, K$$

- Let noise w_k be uniform over $[-U, U]$.
- Local message functions are **binary.**

$$m_k(x_k) = \begin{cases} 1, & x_k \in S_k \\ 0, & x_k \notin S_k \end{cases}$$

where $S_k = \mathbb{R}^+$.

- An ε **-estimator** of θ: $\hat{\theta} = \Gamma(m_1, \cdots, m_K)$ with

$$E\left(\left|\hat{\theta} - \theta\right|^2\right) \leq \varepsilon$$

- We have $P(m_k = 1) = \frac{U + \theta}{2U}$, $P(m_k = 0) = \frac{U - \theta}{2U}$,

$$E(m_k) = \frac{U + \theta}{2U}, \quad \forall k = 1, \cdots, K$$

- Choose the final fusion function Γ as

$$\hat{\theta} = \Gamma(m_1, \cdots, m_K) = -U + \frac{2U}{K} \sum_{k=1}^{K} m_k$$

$$E(\hat{\theta}) = \theta \ , \quad E(\hat{\theta} - \theta)^2 \le U^2 / K$$

- Requiring a total of $K = U^2/\varepsilon$ sensors to compute an ε -estimator for θ

Example II: A decentralized estimation scheme (DES) for the known sensor variances case

- Assume that fusion center knows sensor noise variances [illegible] .
- Let [illegible] and write [illegible] where [illegible] and [illegible] are the integer and decimal parts of [illegible] .
- Construct sensor k 's message function as

[illegible]

where [illegible] is a 0–1 binary random variable with

[illegible]

- The final estimator of at the fusion center is:

$$[illegible]$$

- [illegible] is an unbiased estimator of [illegible], and the MSE

$$[illegible]$$

Moving Horizon State Estimation

Consider a joint density over N states:

$$\left\{x^*(n-N+1),\cdots,x^*(n)\right\} = \arg\max_{\{x(m)\}_{m=n-N+1}^{n}} p\left(x(n-N+1),\cdots,x(n) \mid y(0),\cdots,y(n)\right)$$

Equivalent optimization:

$$\min_{x(n-N),\{w(m)\}_{m=n-N}^{n-1}} \sum_{m=n-N}^{n-1} \left(\|w(m)\|^2_{Q^{-1}} + \|v(m)\|^2_{R^{-1}}\right) + Z_{n-N}\left(x(n-N)\right)$$

prior information
arrival cost

subject to model:

$$x(n+1) = f\left(x(n)\right) + w(n)$$

$$y(n) = h\left(x(n)\right) + v(n)$$

$$w(n) \in W, v(n) \in V, x(n) \in X$$

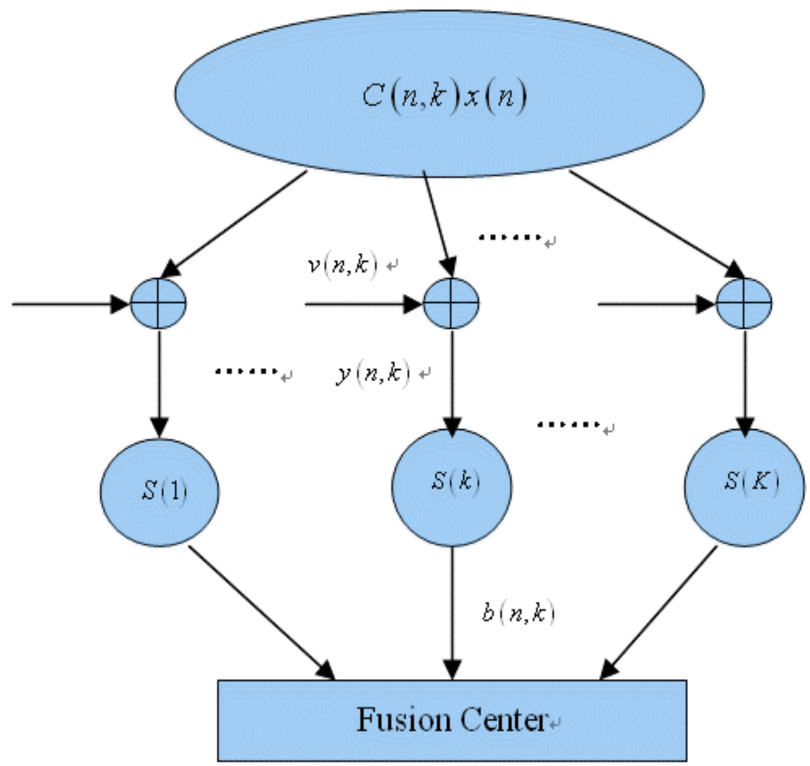
What can moving horizon state estimation do?

Linear model Gaussian noise <u>Stability</u>	Kalman	nonlinear model Gaussian noise	extended Kalman
Linear model General noise <u>Inequality constraints</u> <u>stability</u>	MHE	nonlinear model general noise <u>inequality constraints</u> <u>stability</u>	MHE

online "solution" to quadratic/nonlinear program

Problem Statement

- Let us consider a WSN with a fusion center
- [redacted] distributed sensors [redacted] deployed with the objective of tracking a real random vector $x(n) \in \mathbb{R}^{p\times 1}$



- The underlying system and sensor nodes observing processes can be modeled as

$$x(n) = \mathrm{A}(n)x(n-1) + w(n) \qquad n = 0, 1, 2, \cdots$$

$$y(n,k) = C(n,k)x(n) + v(n,k) \qquad k = 1, \cdots, K$$

where [illegible], [illegible], [illegible], [illegible],

[illegible], [illegible], [illegible],

[illegible], [illegible]

Assumption

- This communication takes place over the shared wireless channel that we will assume can afford transmission of a single packet per time slot .
- Leading to a one-to-one correspondence between time n and sensor index t and allowing us to drop the sensor index t.
- Which sensor is active at time n depends on the underlying scheduling algorithm.

Quantization

- Observation model:

$$y(n) = C(n)x(n) + v(n)$$

- Define a set of thresholds $\{\tau_i, i \in \square\}$, $\tau = \tau_i - \tau_{i-1}$

$$b_i(n) = \begin{cases} 1 & y(n) > \tau_i \\ 0 & y(n) \le \tau_i \end{cases}$$

- Each [illegible] is a Bernoulli random variable with parameter

$$q_i(g(n)) = \Pr\{b_i(n) = 1\} = F(\tau_i - C(n)x(n)) = F(\tau_i - g(n)) \quad i \in \square$$

where [illegible]

- Sensor obtains total binary observation at time n:

  [illegible]

- Without loss of generality, we will assume that [illegible], when [illegible], [illegible] can only take on realizations [illegible] when [illegible]

Main results

- If there is no quantization, we can obtain the estimate of state by

  [illegible]

- Methods
  - Batch filter
  - Kalman filter
  - Moving horizon estimate
  - Particle filter etc.

State estimation using quantized observations

- Consider a joint density over N states:

[illegible]

- Approximate optimization:

[illegible]

where

[illegible]

- **Online "solution" to nonlinear program**

- If [illegible] , then [illegible] , the thresholds are [illegible] , the binary observations are [illegible]
- Approximate optimization is :

[illegible]

[illegible]

Solution steps of moving horizon state estimation for WSN

- Initialization
- At time n ⟶ [illegible], by using state transfer equation [illegible], obtain [illegible].
- Only [illegible] reserve as final estimate at time [illegible].
- At time n+1, obtain [illegible]
- Only [illegible] reserve as final estimate at time [illegible].

Matlab toolbox

- Unconstrained nonlinear programming
  - halver method
  - Newton method
  - Choose functions of MATLAB optimization tool box
    - One variable： fminbnd, fminsearch, fminunc
    - Multi-variable： fminsearch, fminunc

- Constrained nonlinear programming
  - penalty function method
  - Choose functions of MATLAB optimization tool box
    - Multi-variable ： fmincon

The application of Multi-bit DMHE in target tracking

- Consider [illegible] sensors randomly and uniformly deployed in a square region of 100*100 meters and suppose that sensor positions are known

[illegible]

[illegible]

- New observation equation obtained by DES method

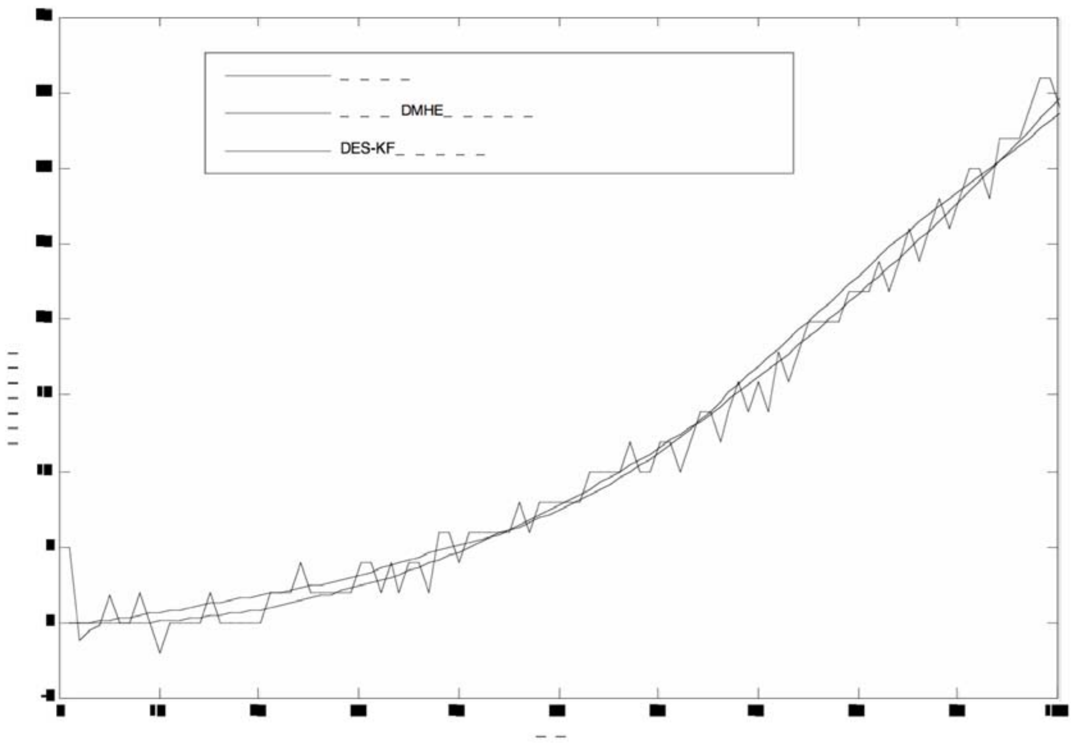
[illegible]

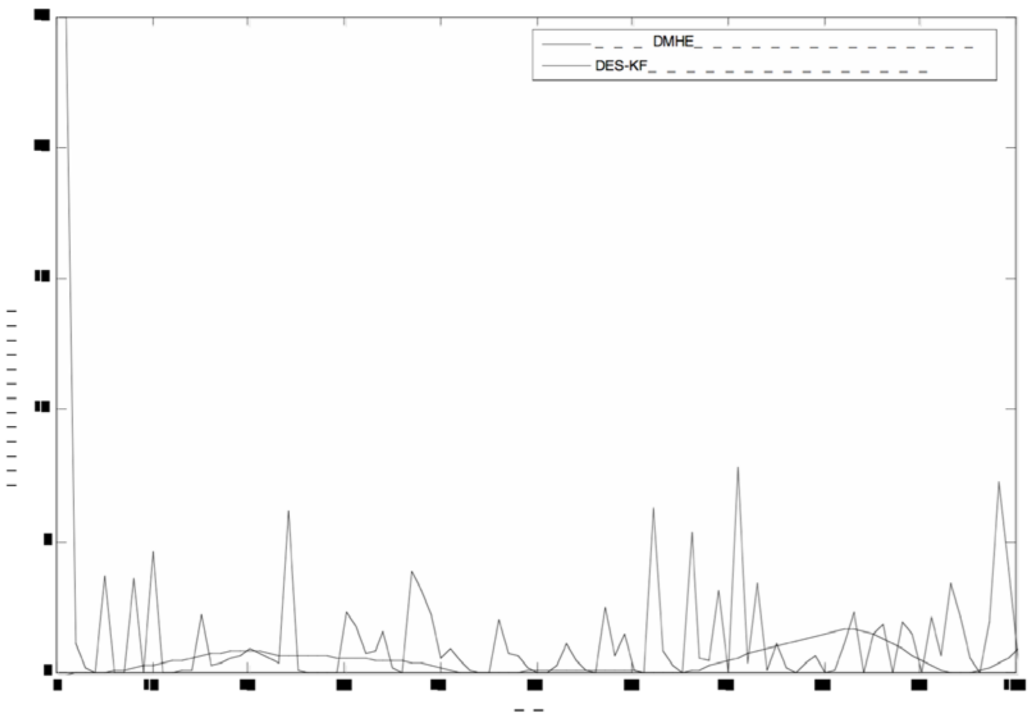
[illegible] with distribution [illegible]

- Suppose only one sensor node works at a time slot, then the observations are given by

[illegible]

Items / Methods	The number of sensors sending information	The most bits transmitted by one node		Threshold changes or not	Estimation performance $J=\sum_{n=1}^{100}\left(x(1,n)-\hat{x}(1,n)\right)^2$
DES-KF	K=1	5.9149		No	147.4031
	K=5	5.9149		No	34.0500
	K=10	5.9149		No	15.7701
Multi-bit DMHE (N=3)	K=1	$\tau=1$	6.6439	Yes	38.0754
		$\tau=0.8$	6.9149	Yes	18.3789
		$\tau=0.5$	7.6439	Yes	12.1556


DMHE
DES-KF


DMHE
DES-KF

Conclusion

- An important problem of wireless sensor network signal processing is that bandwidth is limited, quantization of observations is necessary.
- A dynamic state estimation problem in the context of WSN has been considered.
- Develop a moving horizon state estimation method based on several quantized data for WSN.

Thanks!